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A feedback control problem in Banach spaces

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§ 1. INTRODUCTION. In this note we will consider the following semilinear feedback control system

$$\begin{cases} y_i'(t) = A_i(t)y_i(t) + f_i(t, y_1(t), .., y_n(t), u(t)), \ y_i(t) \in X_i, \ t \in [0,1], \ i = 1, .., n; \\ u(t) \in V(t, y_1(t), .., y_n(t)) \subset Z, \ t \in [0,1], \end{cases} \tag{1}$$

where X_i, $i = 1, \ldots, n$, and Z are Banach spaces. (The spaces X_i are assumed to be separable). Consider the Banach spaces $X = X_1 \times \ldots \times X_n$ and $Y = C([0,1], X_1) \times \ldots \times C([0,1], X_n)$ equipped with the norm $\| z \| = \| (z_1, \ldots, z_n) \| = \max\limits_{1 \le i \le n} \| z_i \|_i$, where $\| \cdot \|_i$ denotes the norm in the space X_i or in $C([0,1], X_i)$ respectively.

Given an acyclic compact set $\mathcal{K} \subset X$, our goal is to provide conditions on A_i, f_i, V and on F_i, defined by $F_i(t, x_1, \ldots, x_n) = f_i(t, x_1, \ldots, x_n, V(t, x_1, \ldots, x_n))$, which guarantee the existence of a mild solution $y(t) = (y_1(t), \ldots, y_n(t))$, $t \in [0,1]$, of (1) with corresponding control $u(t) \in V(t, y_1(t), \ldots, y_n(t))$, $t \in [0,1]$, which satisfies the boundary condition

$$y(1) - y(0) \in \mathcal{K}. \tag{2}$$

Theorem 2.1 below solves the problem. Its proof is based on an abstract existence result (Theorem 2.3 of [4]) concerning the solvability of inclusions in Banach spaces. In fact, in Section 2, we first convert the nonlinear boundary value control problem (1)-(2) into an equivalent system of inclusions

$$\begin{cases} y \in \overline{\Gamma}(x, y) \\ x \in \overline{G}(x, y) \end{cases}$$

where $\overline{\Gamma}$ and $\overline{G}$ are suitable multivalued maps associated to (1) and (2) respectively, $y \in Y$ is the trajectory $y(t)$, $t \in [0,1]$, and $x \in X$ is its initial condition $y(0)$. Then we solve the first inclusion with respect to y considering x as a fixed parameter in a convenient ball $\overline{B}(0,r) \subset X$. Replacing the corresponding solution set $S(x)$ to y in the second equation we obtain a multivalued map $\overline{T} : \overline{B}(0,r) \subset X \multimap X$, given by $\overline{T}(x) = \overline{G}(x, S(x))$, and we look for its fixed points which are the solutions of the nonlinear boundary value control problem (1)-(2).

§ 2. **MAIN RESULT.** We consider (1)-(2) under the following assumptions.

A) For any $i = 1, \ldots, n$, $\{A_i(t)\}_{t\in[0,1]}$ is a family of closed, linear, not necessarily bounded operators in X_i generating strongly continuous evolution operators $U_i : \Delta \to \mathrm{L}(X_i)$, where $\Delta = \{(t,s) \in [0,1] \times [0,1] : 0 \le s \le t \le 1\}$. It is supposed also that the U_i are continuous with respect to the norm of $\mathrm{L}(X_i)$ when $s < t$.

Moreover, they satisfy the following estimates

U1) $\| U_i(t,s) \|^{(\chi)} \le e^{-\delta_i(t-s)}$, where $\delta_i > 0$, $i = 1, \ldots, n$, and $\| B \|^{(\chi)} = \chi_i(BS)$ with $B \in L(X_i)$, S the unit ball in X_i and χ_i the Hausdorff measure of non-compactness in the space X_i, $i = 1, \ldots, n$.

U2) $L_i = \sup\limits_{t,s\in\Delta} \| U_i(t,s) \| < 1, \quad i = 1, 2, \ldots, n.$

Let Z be the Banach space in which the controls take values. We assume that

f) $f_i : [0,1] \times X_1 \times \ldots \times X_n \times Z \to X_i$ is a continuous map, $i = 1, \ldots, n$;

V) the feedback multivalued map $V : [0,1] \times X_1 \times \ldots \times X_n \multimap Z$ has compact values,is u.s.c. and sends bounded sets into bounded sets.

F1) $F_i(t, x_1, \ldots, x_n) := f_i(t, x_1, \ldots, x_n, V(t, x_1, \ldots, x_n))$ is convex for every $(t, x_1, \ldots, x_n)$ $\in [0,1] \times X_1 \times \ldots \times X_n$, $i = 1, \ldots, n$;

F2) $F_i(t, x_1, \ldots, x_n) \subset W_i$ for every $(t, x_1, \ldots, x_n) \in [0,1] \times X_1 \times \ldots \times X_n$, where W_i is a weakly compact, convex subset of X_i, $i = 1, \ldots, n$;

F3) there exists an $(n \times n)$-matrix $\mathcal{M}$ with non-negative components (m_{ij}) such that $\chi_i(F_i(t, D_1, \ldots, D_n)) \le \sum\limits_{j=1}^{n} m_{ij}\chi_j(D_j)$ for every $t \in [0,1]$ and bounded $D_j \subset X_j$.

Let us observe that from conditions f), V), F1) and F2) it follows that the multivalued maps F_i, $i = 1, \ldots, n$, are u.s.c. and that for any functions $y_i \in C([0,1], X_i)$, $i = 1, \ldots, n$, the sets of integrable selections $S^1_{F_i(\cdot, y_1(\cdot), \ldots, y_n(\cdot))}$ are nonempty. Now, let $\mathcal{M} = (m_{ij})_{i,j=1}^{n}$ be the matrix given in F3) and $\mathcal{D}$ be an $n \times n$ diagonal matrix with the coefficients $\delta_1, \ldots, \delta_n$ of the condition U1) on the diagonal. Assume

N) The zero solution of the n-dimensional linear system $z'(t) = (\mathcal{M} - \mathcal{D})z(t)$ is exponentially stable.

We will consider mild solutions of problem (1)-(2), i.e. vectors $(y_1, \ldots, y_n, u) \in C([0,1], X_1) \times \ldots \times C([0,1], X_n) \times L^\infty([0,1], Z)$ such that

$$y_i(t) = U_i(t,0)x_{i0} + \int_0^t U_i(t,s) f_i(s, y_1(s), \ldots, y_n(s), u(s))\, ds$$

$i = 1, \ldots, n$ and $u(s) \in V(s, y_1(s), \ldots, y_n(s))$.
Let us consider the multioperator $\overline{\Gamma} : X \times Y \multimap Y$ defined as

$$\overline{\Gamma}(x,y) = \overline{\Gamma}(x_1, \ldots, x_n, y_1, \ldots, y_n) =$$

$$= \{y \in Y \,:\, y_i(t) = U_i(t,0)x_i + \int_0^t U_i(t,s) g_i(s)\, ds,\ g_i(\cdot) \in S^1_{F_i(\cdot, y_1(\cdot), \ldots, y_n(\cdot))}\}\,.$$

By the Filippov's implicit function lemma (see e.g. [1]) it follows that for any $x \in X$ the set $S(x) = \{y \in Y \,:\, y \in \overline{\Gamma}(x,y)\}$ coincides with the set of mild solutions $(y_1(t), \ldots, y_n(t))$, $t \in [0,1]$, of system (1) with initial condition $x = (x_1, \ldots, x_n)$. Following the methods of [3] one can prove that the multioperator $\overline{\Gamma}$ is closed. Now, let us define in the spaces $C([0,1], X_i)$, $i = 1, \ldots, n$, the following measure of non-compactness

$$\phi_i(D) = \sup_{t \in [0,1]} \chi_i(D(t) e^{-rmt})$$

where $D(t) = \{y(t) \,:\, y \in D\}$, $m = \max\{m_{ij},\ 1 \le i \le n,\ 1 \le j \le n\}$ *and* $r > n$ is a fixed number. Let

$$\psi_Y(\Omega) = \max_{1 \le i \le n} \phi_i(\Omega_i)\,, \quad \text{where } \Omega_i = \mathrm{Proj}_{Y_i}(\Omega),\ \Omega \subset Y\,.$$

It results that ψ_Y is a monotone, non-singular measure of non-compactness. We have the following Lemma.

Lemma 2.1 *The multioperator* $\overline{\Gamma}$ *is* ψ_Y*-condensing in the second variable.*

As a consequence of N) the linear operator $B = e^{\mathcal{M} - \mathcal{D}} : \mathbf{R}^n \to \mathbf{R}^n$ has the spectral radius less than 1. Let l be such that $\| B^l \| < q^l$, where $q < 1$. Let $\chi : 2^X \to (\mathbf{R}^n_+, \ge)$ be a vector valued measure of non-compactness in X, defined as $\chi(\Omega) = \{\chi_1(\Omega_1), \ldots\ldots, \chi_n(\Omega_n)\}$. Then $\psi_X(\Omega) : 2^X \to \mathbf{R}_+$ defined as

$$\psi_X(\Omega) = q^{l-1} \| \chi(\Omega) \| + q^{l-2} \| B\chi(\Omega) \| + \ldots + \| B^{l-1} \chi(\Omega) \|$$

is a monotone, non-singular measure of non-compactness in X. Consider now the multioperator $\overline{G} : X \times Y \multimap X$ defined as

$$\overline{G}(x,y) = y(1) - \mathcal{K}\,,$$

and the map $\overline{T} : X \multimap X$ given by $\overline{T}(x) = \overline{G}(x, S(x))$. Following [2], we can state next Lemma.

Lemma 2.2 *The multivalued map $\overline{T}$ is ψ_X-condensing.*

Furthermore, there exists a ball centered at the origin with radius r, $\overline{B}(0,r) \subset X$, such that the multivalued operator $\overline{T}$ satisfies the following condition: if $\lambda x \in \overline{T}(x)$, with $x \in \partial B(0,r)$ and $\lambda \in \mathbf{R}$, then $\lambda \leq 1$ (called property "P"). In fact, the next Lemma holds.

Lemma 2.3 *The multivalued map $\overline{T}$ satisfies the condition $\overline{T}(\partial B(0,r)) \subset B(0,r)$.*

Finally, we can prove the following result.

Theorem 2.1 *Assume A), U1), U2), f), V), F1)-F3) and N) then there exists a mild solution $y(t), t \in [0,1]$, of the nonlinear boundary value control problem* (1)-(2).

The proof consists in verifying, by using the previous lemmas and the topological properties of the sets $S(x)$ (see [2],[5]), that all the assumptions of the following existence result are satisfied.

(**Theorem 2.3** of [4]) *Let X, Y be Banach spaces and let ψ_X and ψ_Y be monotone non-singular measures of non-compactness in X and Y respectively. Let $\overline{\Gamma} : X \times Y \multimap Y$ be a closed multivalued map ψ_Y-condensing in the second variable and let $\overline{G} : X \times Y \multimap X$ be an admissible multivalued map. Let us suppose that for every $x \in X$ the set $S(x) = \{y \in Y : y \in \overline{\Gamma}(x,y)\}$ is nonempty and acyclic. Let the multivalued map $\overline{T} : \overline{B}(0,r) \subset X \multimap X$, defined by $\overline{T}(x) = \overline{G}(x, S(x))$, be ψ_X-condensing and satisfying the property "P" on the boundary $\partial B(0,r)$. Then the system of inclusions*

$$\begin{cases} y \in \overline{\Gamma}(x,y) \\ x \in \overline{G}(x,y) \end{cases}$$

has a solution.

Remark 2.1. If $\mathcal{K} = \{0\}$ then we have solved the periodic problem $y(0) = y(1)$.

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