12TH WORLD CONGRESS
INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL

Sydney, Australia
18 - 23 July 1993

IFAC

PREPRINTS OF PAPERS

VOLUME 5

Sponsored by the International Federation of Automatic Control

A sliding manifold approach to satellite attitude control

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Keywords—Attitude control; satellite control; stabilization; nonlinear systems; singularly perturbed systems.

Abstract

A sliding manifold based control strategy is proposed for the attitude control of a satellite. By using the theory of singular perturbation a PD feedback control is designed. It exhibits robustness properties with respect to environmental disturbances and plant parametric uncertainties. The resulting control signal remains, after a fast transient, in a neighbourhood of the well-defined equivalent control. Then the control can take into account bounds on the available control signals and avoid the peaking phenomenon of high gain systems. Finally the proposed procedure is applied to the CARINA capsule, and simulations are performed by using the ESA-MIDAS dynamic simulator in presence of environmental disturbances and corrupted Earth-magnetic field measures.

1 INTRODUCTION

Motion of a spacecraft presents two dynamical aspects of interest. The most obvious one is the trajectory traced by its center of mass which is governed by the classical Keplerian relations. On the other side, a satellite, while following a trajectory, may execute rotational motion about its center of mass, commonly referred to as *libration*.

There are numerous situations of practical importance such as communications, scanning of cloud cover for weather forecasting, survey of Earth resources, scientific and military observations, etc., where it is desirable to mantain a satellite in a fixed orientation with respect to the Earth. Unfortunately, even though a spacecraft may be precisely oriented at launch, it tends to deviate from this preferred orientation under the influence of environmental forces as solar pressure, interactions with the Earth's gravitational and magnetic fields and, if the satellite happens to be close to the Earth, free molecular reaction forces. Internal motion of payload, astronauts and sloshing propellant as well as coupling of attitude dynamics with the orbital and flexural mechanics may add to the problem. This leads to undesirable libration motion which must be controlled for successful completetion of a given mission.

Several methods of attitude control have been developed over the years. Broadly speaking, they may be classified as active and passive techniques.

For missions where pointing accuracy requirements are not too exacting, passive techniques involving no expenditures of stored energy have proven to be adequate. Passive stabilization techniques are generally achieved by designing satellites with physical characteristics (such as booms; flaps like aileron, elevator and rudder of an airplane; magnetic dipoles, etc.) which interact with the environmental forces in a manner so as to maintain a specified orientation.

For attainment of high pointing accuracy active techniques must be adopted. The actuators typically used in active control are micro-thruster units, magnetic coils and wheels. In particular all the missions performed in low orbit and Earth-pointing requiring a three axes stabilization can be controlled by means of two magnetic coils and a reaction wheel. This kind mission is dealt with in this paper. The main problem of these techniques is that energy supplies are very expensive commodities on a satellite, so bounds on the available control have to be taken into account.

The life-cycle of the mission can be split into the following phases:

- Launch
- Operative attitude acquisition
- Operative phase.

Moreover, for re-entry vehicle

- Re-entry
- Recovery.

We address the attitude stabilization problem during the operative phase. In addition to the attitude problems mentioned above in this phase the control has to recover residual angular rates due to the manoeuvers in the previous phase executed by means of thrusters.

The problem has been the subject of many works (Columbia Astrophysics Laboratory, 1988), (Collins and Bonello, 1973), (Cavallo *et alii*, 1992), (Koenigsmann *et alii*, 1991). In (Byrnes and Isidori, 1990) the problem of achieving a desired attitude by means of two actuators is examined and a nonlinear feedback control law is proposed. In (Wen and Kreutz, 1991) by using a Lyapunov approach a proportional–derivative feedback law plus some feedforward is proposed.

The control policy we propose is based on a sliding manifold approach. It has been previously developed in (Cavallo *et alii*, 1992) in the case of a tracking problem for a rigid robot. Specifically, given the initial configuration of the system and bound on the control torques, we can design a sliding manifold for which the equivalent control is well defined. Then, by using the singular perturbation theory, a PD feedback controller is designed, which turns out to be the solution of a system of differential equations containing a small positive parameter. The resulting control law assures that the control signal, after a rapid transient, remains close to the equivalent control in the uniform topology. Then the closed-loop system exhibits stable behaviour and insensitivity with respect to disturbances and variations of the satellite parameters avoiding the drawbacks of high gain systems such as peaking phenomena that, due to the control limitations described above, are not admissible for the spacecraft actuators. Moreover, the system state is in a prescribed neighbourhood of the sliding manifold at any time. The proposed control algorithm is then applied to a possible configuration of the CARINA satellite. CARINA (see Fig. 1) is a new retrievable unmanned capsule developed by ALENIA SPAZIO under ASI contract for micro-gravity experiments. Its operative phase has a nominal duration of five days, with the capsule in a circular Low Earth Orbit (300 km) nearly equatorial (inclination 2.9°). The attitude control is devoted to communication link purposes and to put the capsule nose in upwind direction as to obtain a stabilizing effect from the aerodynamic torques due to the capsule shape. Simulations are performed by using the ESA-MIDAS dynamic simulator in presence of athmospheric disturbances and corrupted Earth-magnetic field measures.

2 THE MODEL

In this section the model of the satellite will be derived. To this end we must first introduce the reference frames usually used in a satellite attitude control problem and then the dynamics equations.

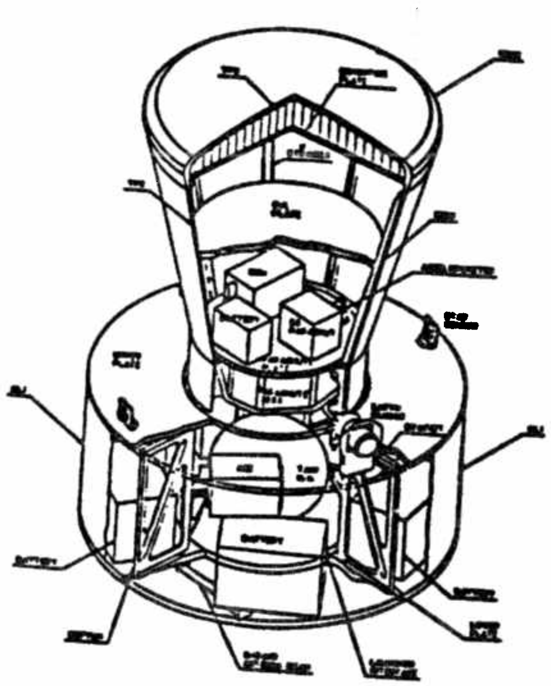

Figure 1: CARINA re–entry capsule

2.1 The reference frames

The coordinate systems used in attitude control are shown in Fig. 2. The inertially fixed coordinate

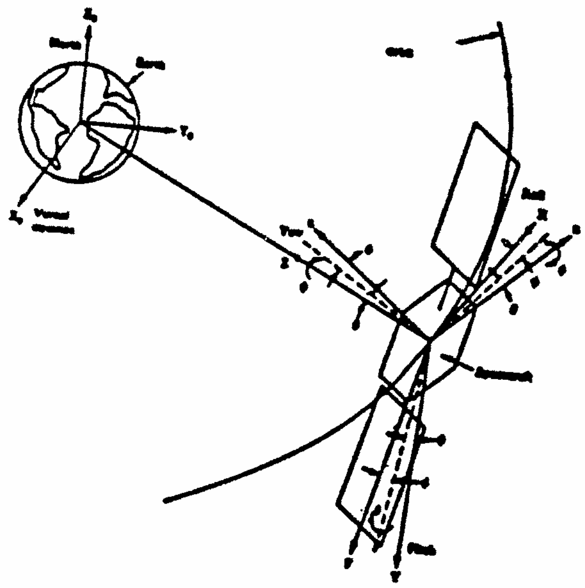

Figure 2: Reference frames

system X_0, Y_0, Z_0 with the axes origin in the Earth center is used to determine the orbital position of the satellite. The attitude motion of a spacecraft is most commonly described in terms of an "airplane" three axis coordinate system, namely roll, pitch and yaw.

The nominal yaw axis, Z, is along the vector from the center of mass of the spacecraft to the center of mass of the Earth, and the nominal pitch axis Y is normal to the orbit plane in such a way that the XYZ coordinate system is a right handed mutually orthogonal frame. The coordinate system XYZ is also called the orbit coordinate system. The origin of the coordinate system is at the center of mass of the spacecraft. It is rotating about the Y axis with respect to the inertially fixed system X_0, Y_0, Z_0 at the orbital rate of ω_0.

The last reference system used is the body fixed reference frame xyz, that, for the sake of simplicity, is taken coincident with the spacecraft central axes of inertia.

The perturbed attitude of the spacecraft fixed coordinate system xyz is obtained from the nominal attitude by the following rotations: ψ about the Z axis, θ about the once-displayed Y axis, and ϕ about the twice-displayed X axis. The angles ψ, θ, ϕ are called yaw, pitch and roll errors, respectively.

2.2 Dynamics equations

The rigid body dynamics equations with respect to the principal axes of inertia can be written as

$$\begin{aligned} M_x &= I_{xx}\dot{\omega}_x + \omega_y\omega_z(I_{zz} - I_{yy}) \\ M_y &= I_{yy}\dot{\omega}_y + \omega_x\omega_z(I_{xx} - I_{zz}) \\ M_z &= I_{zz}\dot{\omega}_z + \omega_x\omega_y(I_{yy} - I_{xx}) \end{aligned} \quad (1)$$

where ω_x, ω_y and ω_z are the angular velocities, M_x, M_y and M_z the applied torques and I_{xx}, I_{yy} and I_{zz} the momenta of inertia.

Introducing the attitude error angles ϕ, θ and ψ, which are known as roll, pitch and yaw errors respectively, the angular velocities can be expressed as (Kaplan, 1976), (Wertz, 1986)

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\theta & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} - \omega_0 \begin{pmatrix} \cos\theta\sin\psi \\ \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi \\ -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi \end{pmatrix} \quad (2)$$

where ω_0 is the orbital rate. It is known that such a description is valid only when $\theta \in (-\pi/2, \pi/2)$ and $\phi \in (-\pi/2, \pi/2)$.

Moreover the vector of applied torques can be split into two components

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \mathbf{M}_a + T_g \quad (3)$$

where $\mathbf{M}_a$ is the vector of torque due to the actuators and

$$T_g = 3\omega_0^2 \begin{pmatrix} (I_{zz} - I_{yy})\sin\phi\cos\phi\cos^2\theta \\ (I_{zz} - I_{xx})\sin\theta\cos\theta\cos\phi \\ (I_{xx} - I_{yy})\sin\theta\cos\theta\sin\phi \end{pmatrix} \quad (4)$$

is the vector of gravity gradient torques.

Denote by $\boldsymbol{\xi} = (\phi, \theta, \psi)^T$ the vector of **error angles**. Then eqn (1) can be rewritten as

$$\mathbf{J}(\boldsymbol{\xi})\ddot{\boldsymbol{\xi}} + \left(C(\boldsymbol{\xi})Z(\dot{\boldsymbol{\xi}}) + \mathbf{B}(\boldsymbol{\xi})\right)\dot{\boldsymbol{\xi}} + \mathbf{G}(\boldsymbol{\xi})\boldsymbol{\xi} = \mathbf{M}_G(\boldsymbol{\xi})\boldsymbol{\xi} + \mathbf{M}_a \quad (5)$$

where

$$\mathbf{Z}^T(\dot{\boldsymbol{\xi}}) = \begin{pmatrix} \dot{\phi} & 0 & 0 & 0 & 0 & 0 \\ 0 & \dot{\phi} & 0 & \dot{\theta} & 0 & 0 \\ 0 & 0 & \dot{\phi} & 0 & \dot{\theta} & \dot{\psi} \end{pmatrix} \quad (6)$$

and the *total inertia matrix* is

$$\mathbf{J}(\boldsymbol{\xi}) = \begin{pmatrix} I_{xx} & 0 & -I_{xx}\sin\theta \\ 0 & I_{yy}\cos\phi & I_{yy}\cos\theta\sin\phi \\ 0 & -I_{zz}\sin\phi & I_{zz}\cos\theta\cos\phi \end{pmatrix} \quad (7)$$

Note that such a matrix, when $\theta \in (-\pi/2, \pi/2)$ and $\phi \in (-\pi/2, \pi/2)$, has all its eigenvalues in the right half complex plane.

The remaining matrices can be expressed via standard algebra. In particular the matrix $C(\boldsymbol{\xi})$ takes into account Coriolis and centrifugal effects, $\mathbf{B}(\boldsymbol{\xi})$ torques depending on gyroscopic effects, $\mathbf{G}(\boldsymbol{\xi})$ torques depending on the attitude and $\mathbf{M}_G(\boldsymbol{\xi})$ gravity-gradient torques.

Rearranging eqn (5) in terms of the state vector $x = (\boldsymbol{\xi}^T, \dot{\boldsymbol{\xi}}^T)^T$ we obtain

$$\dot{x} = \begin{pmatrix} 0 & \mathbf{I} \\ \mathbf{J}^{-1}(\boldsymbol{\xi})(\mathbf{M}_G(\boldsymbol{\xi}) - \mathbf{G}(\boldsymbol{\xi})) & -\mathbf{J}^{-1}(\boldsymbol{\xi})\left(\mathbf{C}(\boldsymbol{\xi})\mathbf{Z}(\dot{\boldsymbol{\xi}}) - \mathbf{B}(\boldsymbol{\xi})\right) \end{pmatrix} x + \begin{pmatrix} 0 \\ \mathbf{J}^{-1}(\boldsymbol{\xi}) \end{pmatrix} \mathbf{M}_a \quad (8)$$

The expression of $\mathbf{M}_a$ depends on the choice of the actuators.

By using two magnetic coils, the first along the x axis, the second along the z axis and a reaction wheel whose spin axis is aligned with the y axis, $\mathbf{M}_a$ can be written as (Wertz and Larson, 1991)

$$\mathbf{M}_a = \begin{pmatrix} 0 & -B_y & 0 \\ -B_z & B_x & 1 \\ B_y & 0 & 0 \end{pmatrix} \begin{pmatrix} \mu_x \\ \mu_z \\ \dot{h} \end{pmatrix} + \begin{pmatrix} 0 & 0 & -h \\ 0 & 0 & 0 \\ h & 0 & 0 \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \quad (9)$$

where $(B_x, B_y, B_z)^T$ is the Earth magnetic field expressed in body axes, h is the angular momentum of the wheel and μ_x, μ_z are the components of the vector of dipole momenta in body axes on the x and z-axes respectively.

Under the usual assumption to neglect the wheel dynamics, the input-output behaviour of the wheel is described by $h = K_v V$ where V is the control voltage of the wheel and K_v is the wheel gain. Then eqn. (8) can be rewritten as

$$\dot{z} = \begin{pmatrix} 0 & \mathbf{I} & 0 \\ \mathbf{J}^{-1}(\xi)(\mathbf{M}_G(\xi) - \mathbf{G}(\xi)) & -\mathbf{J}^{-1}(\xi)\left(C(\xi)\mathbf{Z}(\xi) - \mathbf{B}(\xi)\right) & \mathbf{D}(\xi, \dot{\xi}) \\ 0 & 0 & 0 \end{pmatrix} z + \begin{pmatrix} 0 \\ \mathbf{J}^{-1}(\xi) T_{WB}(\xi) \\ 0 \quad 0 \quad K_v \end{pmatrix} u \tag{10}$$

where $z = (x^T, h)^T$ and $u = (\mu_x, \mu_z, V)^T$,

$$T_{WB}(\xi) = \begin{pmatrix} 0 & -B_y & 0 \\ -B_z & B_x & K_v \\ B_y & 0 & 0 \end{pmatrix} \tag{11}$$

and the vector

$$\mathbf{D}(\xi, \dot{\xi}) = \begin{pmatrix} \dfrac{\omega_z}{I_{xx}} + \dfrac{\omega_x \tan\theta\cos\phi}{I_{zz}} \\ -\dfrac{\omega_x \sin\phi}{I_{zz}} \\ \dfrac{\omega_x \cos\phi}{I_{zz}\cos\theta} \end{pmatrix} \tag{12}$$

expresses the interaction between the angular momenta of the wheel and the capsule. Note that the matrix T_{WB} depends only on the vector ξ. Moreover, the entries of this matrix can be measured by means of magnetometers on the capsule.

3 THE ATTITUDE CONTROL

Let $\mathcal{R}$ be the open connected subset of R^6 in which the state variables x of system (10) take the values. Observe that $0 \in \mathcal{R}$.

In this section we address the problem of regulating x to zero. This regulation problem can be stated as follows.

Given $\beta > 0$, $\delta > 0$ and $x_0 \in \mathcal{R}$, it is required to design a feedback control law such that the solution $z(t) = (x(t)^T, h(t))^T$ of system (10) with $z(0) = (x_0^T, h_0)^T$ satisfies

$$| x(t) | \leq \delta + Ae^{-\beta t} \tag{13}$$

for any $t \in [0, T]$, where A is a constant depending on the data and $T = 5$ days (the nominal duration of the operative phase).

To solve this problem we must introduce some preliminaries.

3.1 Mathematical preliminaries

System (10) can be rewritten in the following form

$$\dot{z} = A(z)z + B(z)u \tag{14}$$

where $z \in \mathcal{R}_1 = \mathcal{R} \times \mathbf{R}$, $u \in \mathbf{R}^3$ and the maps $z \to A(z)$, and $z \to B(z)$ satisfy a locally Lipschitz condition on $\mathcal{R}_1$. Let $s : \mathcal{R}_1 \times [0, T] \to \mathbf{R}^3$ be a given continuously differentiable map and define a related sliding manifold as

$$\mathcal{S} = \{(z, t) \in \mathcal{R}_1 \times [0, T] : s(z, t) = 0\} \tag{15}$$

Consider the function $g(\mathbf{Z}, u, t) : \mathcal{R}_1 \times \mathbf{R}^3 \times [0, T] \to \mathbf{R}^3$ defined by

$$g(\mathbf{Z}, u, t) = \frac{\partial}{\partial t}s(z, t) + \frac{\partial}{\partial z}s(z, t)\,(A(z)z + B(z)u) \tag{16}$$

and for any $\varepsilon > 0$ consider the system of ordinary differential equations

$$\dot{z} = A(z)z + B(z)u \tag{17}$$
$$\varepsilon\dot{u} = g(\mathbf{Z}, u, t) \tag{18}$$

Now we make the following assumptions on $\mathcal{S}$

S1 For any $t \in [0, T]$ there exists $z \in \mathcal{R}_1$ such that $(z, t) \in \mathcal{S}$.

S2 There exists a neighbourhood $\mathcal{I}$ of the manifold $\mathcal{S}$ such that, for any $(z, t) \in \mathcal{I}$, the map $u \to g(x, u, t)$ is one–to–one and its range contains zero.

S3 For any pair $(z_0, u_0) \in \mathcal{R}_1 \times \mathbf{R}^3$ and any $\varepsilon > 0$, the system (17)–(18) has an unique solution $(z(t,\varepsilon), u(t,\varepsilon))$ defined on $[0, T]$ such that $(z(0,\varepsilon), u(0,\varepsilon)) = (z_0, u_0)$

S4 The equilibrium point $\bar{u}^* = u^*(\bar{z}, \bar{t})$ of the equation (18) is asymptotically stable, for any $(\bar{z}, \bar{t}) \in \mathcal{I}$. Furthermore, we suppose that the asymptotic stability is uniform in $(\bar{z}, \bar{t}) \in \mathcal{I}$. That is the solution $y = y(s)$ of the Cauchy problem

$$\dot{y} = g(\bar{z}, y, \bar{t})$$
$$y(0) = \bar{u} \tag{19}$$

satisfies $\lim_{s \to \infty} y(s) = \bar{u}^*$, whenever $\bar{u}$ is sufficiently closed to $\bar{u}^*$ and $(\bar{z}, \bar{t}) \in \mathcal{I}$.

Now we give the following

Definition 1 *For any $(z, t) \in \mathcal{I}$, the unique solution $u^*(z, t)$ of the algebraic equation $g(\mathbf{Z}, u, t) = 0$ is called the* equivalent control *for the problem*

$$\dot{z} = A(z)z + B(z)u$$
$$s(z, t) = 0 \tag{20}$$

Referring to (16), it is clear that a necessary and sufficient condition for Assumption **S2** is that the matrix

$$\frac{\partial}{\partial z}s(z, t)B(z) \tag{21}$$

is not singular for any $(z,t) \in \mathcal{I}$. This condition also defines uniquely the equivalent control as

$$u^*(z,t) = -\left(\frac{\partial}{\partial z}s(z,t)B(z)\right)^{-1}\left(\frac{\partial}{\partial t}s(z,t)+\frac{\partial}{\partial z}s(z,t)A(z)z\right). \quad (22)$$

Definition 2 *A point $(\bar{z},\bar{u},\bar{t})$ such that the solution of the Cauchy problem (19) converges asymptotically to $u^*(\bar{z},\bar{t})$ is said to belong to the* domain of influence *of $u^*(\bar{z},\bar{t})$.*

The following Theorem is a direct consequence of a classic result of the Singular Perturbation Theory (Wasow, 1965).

Theorem 1 *Assume that* **S1–S4** *are satisfied. Let $(z_0, u_0, 0)$ be a point in the domain of influence of $u^*(z_0,0)$, with $(z_0,0) \in \mathcal{S}$. Then the solution pair $(z(t,\varepsilon), u(t,\varepsilon))$ of the Cauchy problem*

$$\begin{aligned} \dot{z} &= A(z)z + B(z)u; \quad z(0) = z_0 \\ \varepsilon\dot{u} &= g(\mathbf{z}, u, t); \quad u(0) = u_0 \end{aligned} \quad (23)$$

has the following properties

$$\begin{aligned} \lim_{\varepsilon\to 0} z(t,\varepsilon) &= \tilde{z}(t) \quad \text{uniformly in } [0,T] \\ \lim_{\varepsilon\to 0} u(t,\varepsilon) &= \tilde{u}(t) \quad \text{uniformly in } [t_1,T] \end{aligned} \quad (24)$$

for any $t_1 > 0$ and $(\tilde{z}(t),\tilde{u}(t))$ is the solution of the reduced system

$$\begin{aligned} \dot{z} &= A(z)z + B(z)u; \quad z(0) = z_0 \\ 0 &= g(\mathbf{z}, u, t). \end{aligned} \quad (25)$$

3.2 Feedback control design

In order to solve the attitude control problem we choose a control law u of the form

$$u = \mathrm{T}_{WB}^{-1}(\xi)u' \quad (26)$$

and consequently, in order to design u', we define $s : \mathcal{R}_1 \times [0,T] \to \mathbf{R}^3$ as follows

$$s(z,t) = H\left(-z + e^{\mathrm{C}t}z_0\right) \quad (27)$$

where H and C are matrices of suitable dimensions to be chosen later and $z_0 = (x_0^T, 0)^T$.
Now consider the system

$$\dot{z} = \begin{pmatrix} 0 & \mathrm{I} & 0 \\ \mathrm{J}^{-1}(\xi)(\mathrm{M}_G(\xi) - \mathrm{G}(\xi)) & -\mathrm{J}^{-1}(\xi)\left(C(\xi)\mathrm{Z}(\dot{\xi}) - \mathrm{B}(\xi)\right) & \mathrm{D}(\xi,\dot{\xi}) \\ 0 & 0 & 0 \end{pmatrix} z + \begin{pmatrix} 0 \\ \mathrm{J}^{-1}(\xi) \\ (0 \quad 0 \quad K_v)\mathrm{T}_{WB}^{-1}(\xi) \end{pmatrix} u' \quad (28)$$

$$\varepsilon\dot{u}' = \mathbf{g}(z, u', t) \quad (29)$$

where $\mathbf{g}$ is defined by means of (16) and (27). Since $s(z_0, 0) = \mathbf{0}$, the solution $(\tilde{z}(t), \tilde{u}'(t))$ of the reduced system defined in Theorem 1 satisfies $s(\tilde{z}(t),t) = 0$ for any $t \in [0,T]$.
By Theorem 1 we know that if assumptions **S1–S4** are satisfied, then the solution $z(t,\varepsilon)$ of system (28)–(29) with the corresponding control law

$$u'(t,\varepsilon) = \frac{1}{\varepsilon}H\left(-z(t,\varepsilon) + e^{\mathrm{C}t}z_0\right) \quad (30)$$

approaches $\tilde{z}(t)$ uniformly in $[0,T]$. As we are interested in regulating only the vector $x = (\xi^T, \dot{\xi}^T)^T$ we let

$$H = (H_1 \quad H_2 \quad 0) \quad (31)$$

where H_1 and H_2 are full rank 3×3 constant matrices to be designed. According to (21) it is easy to prove that assumption **S2** holds if and only if the matrix

$$\begin{aligned} \Gamma(\xi) &= H_2\mathrm{J}^{-1}(\xi) \\ \forall \xi \in \mathcal{R}_0 &= \mathcal{R} \cap \{(p^T, 0^T)^T, p \in \mathbf{R}^3\} \end{aligned} \quad (32)$$

is nonsingular. Since, for any $\xi \in \mathcal{R}_0$, the matrix $\mathbf{J}(\xi)$ has all the eigenvalues with positive real part, it is possible to select the matrix H_2 in such a way that for any compact set $\mathcal{K} \subset \mathcal{R}_0$ there exists a constant $\alpha_K > 0$ such that

$$\mathrm{Re}\lambda_{\min}(\Gamma(\xi)) \geq \alpha_K > 0 \quad (33)$$

for each $\xi \in \mathcal{K}$, where $\mathrm{Re}\lambda_{\min}(A)$ denotes the minimum of the real parts of all the eigenvalues of the matrix A. Therefore condition **S4** is satisfied. **S1** is a direct consequence of definition (27). Finally, by the properties of the matrices $\mathbf{A}(\cdot)$ and $\mathbf{B}(\cdot)$ which model the system, it can be derived that system (28)–(29) satisfies a locally Lipschitz condition. Furthermore, by (40) and the estimates (43), (44) and (45) in the sequel, the solution $(z(t,\varepsilon), u'(t,\varepsilon))$ of (28)–(29) for which $(z(0,\varepsilon), u'(0,\varepsilon)) = (z_0, u'_0) \in \mathcal{R}_1 \times \mathbf{R}^3$ is defined on $[0,T]$ if $|z_0|$, and hence $|x_0|$, is sufficiently small. Observe that how small $|z_0|$ has to be depends only on the data $H = (H_1\ H_2)$ and C.
Remember that for a symmetric matrix C, if we choose the Euclidean norm for vectors and the induced norm for matrices, then

$$\lambda_{\max}(C) = \mu(C) \quad (34)$$

where $\mu(\cdot)$ denotes the logarithmic norm, or measure of a matrix, (Coppel, 1978), (Desoer and Vidyasagar, 1985).
Consider system (28)–(29) and assume that the 3×3 matrix H_2 is such that $\Gamma(\xi)$ given by (32) satisfies (33). Now we can state the following

Theorem 2 *Let $\beta > 0$, $\delta > 0$, $\gamma > 0$. Let x_0 be given in such a way that $\tilde{x}(t) \in \mathcal{R}$ for any $t \in [0,T]$. Assume that*

$$\mathrm{Re}\lambda_{\min}(H_2^{-1}H_1) \geq \beta + \gamma \quad (35)$$

and

$$\mu(C) < -\mathrm{Re}\lambda_{\min}(H_2^{-1}H_1) + \gamma \quad (36)$$

then there exists $\varepsilon_0 > 0$ *such that for any* $\varepsilon \in (0, \varepsilon_0]$ *the solution* $(z(t,\varepsilon), u'(t,\varepsilon))$, *with* $z(t,\varepsilon) = (x^T(t,\varepsilon), h(t,\varepsilon))^T$, *to (28)-(29) satisfying* $(z(0,\varepsilon), u'(0,\varepsilon)) = (z_0, u'_0)$ *is such that*

$$| x(t,\varepsilon) | < \delta + A_1 e^{\mu(C)t} + A_2 e^{-\beta t} \quad (37)$$

with

$$u'(t,\varepsilon) = \frac{1}{\varepsilon}(H_1\ H_2\ 0)\left(-z(t,\varepsilon) + e^{Ct}z_0\right) + u'_0 \quad (38)$$

where $t \in [0,T]$ *and* A_1 *and* A_2 *are positive constants depending on* H_1, H_2, C *and* x_0.

Proof. Denote by $(\tilde{z}(t), \tilde{u}'(t))$ the solution of the reduced system associated to (28)–(29), with $\tilde{z}(0) = z_0$. Since $s(\tilde{z}(t),t) = 0$ for all $t \in [0,T]$, we have

$$(H_1\ H_2)\tilde{x}(t) = He^{Ct}z_0. \quad (39)$$

This implies

$$\dot{\tilde{\xi}}(t) = -H_2^{-1}H_1\tilde{\xi}(t) + H_2^{-1}He^{Ct}z_0 \quad (40)$$

so by assumptions on the matrices $H_2^{-1}H_1$ and C we have

$$\lim_{t\to\infty} \tilde{x}(t) = 0 \quad (41)$$

exponentially. Consider now a neighbourhood $\mathcal{I}$ of $\mathcal{S}$ such that the set $\{z \in \mathbf{R}^7 : (z,t) \in \mathcal{I}$ for some $t \in [0,T]\}$ is bounded with respect to the x–variable and $\mathcal{I} \subset \mathcal{R}_1$. For $\varepsilon > 0$ sufficiently small, under our assumptions it is not hard to show that conditions **S1**–**S4** are verified in $\mathcal{I}$. In fact, **S1** is a direct consequence of the definition of s and **S2**, **S4** can be derived from inequality (33). Finally, **S3** holds since $\tilde{z}(t) \in \mathcal{R}_1$ and it is defined on $[0,T]$. Hence by Theorem 1 we get the following inequalities

$$| x(t,\varepsilon) | \le | x(t,\varepsilon) - \tilde{x}(t) | + | \tilde{x}(t) | < \delta + | \dot{\tilde{\xi}}(t) | + | \tilde{\xi}(t) | \quad (42)$$

Finally, using our assumptions, eqn. (40) and (Coppel, 1978), for sufficiently small $\gamma > 0$, we obtain

$$| x(t,\varepsilon) | < \delta + A_1 e^{\mu(C)t} + A_2 e^{(-\mathrm{Re}\lambda_{\min}(H_2^{-1}H_1)+\gamma)t} \quad (43)$$

where, since $|x_0| = |z_0|$, we obtain

$$A_1 = \| H_2^{-1}H \| \, | x_0 | \quad (44)$$

and

$$A_2 = L(1 + \| H_2^{-1}H_1 \|)\left(| \xi(0) | + \frac{\| H_2^{-1}H \| \, | x_0 |}{| \mu(C) + \mathrm{Re}\lambda_{\min}(H_2^{-1}H_1) - \gamma |}\right) \quad (45)$$

and $L = L(\gamma)$ is the constant of Proposition 3 in Coppel. Then (37) can be easily derived.

The choice of the matrix C is a degree of freedom of our design. The role of this matrix is to "drive" the decaying of the state x in the sense that the state tends to track the term $e^{Ct}x_0$. Then the closer to zero the real part of the eigenvalues of C, the slower the convergence of the state to zero will be. This allows us to consider bounds on the control: by Theorem 1 the control signal $u'(t,\varepsilon)$ approaches the equivalent control $\tilde{u}'(t)$ on $[t_1,T]$ and the shape of the equivalent control is determined by the matrix C according to (22). Then we can look for the matrix C minimizing a suitable performance index; this topic is the subject of work in progress. Moreover the presence of the term $e^{Ct}x_0$ avoids the drawbacks of high gain control systems such as peaking phenomena.

4 SIMULATION RESULTS

In this section the control law (29) presented in the previous section is used to obtain the attitude control of the CARINA satellite. The simulations have been performed by using the ESA–MIDAS dynamic simulator. Assuming a quasi–circular orbit, i.e. small eccentricity, the data for the satellite and actuators are

$$\begin{aligned} \omega_0 &= \frac{2\pi}{5442} && \text{rad/s} \\ I_{xx} &= 66 && \text{kg}\cdot\text{m}^2 \\ I_{yy} &= 152 && \text{kg}\cdot\text{m}^2 \\ I_{zz} &= 152 && \text{kg}\cdot\text{m}^2 \\ K_v &= 1/1000 && \text{N}\cdot\text{m/V} \end{aligned} \quad (46)$$

The data for the control, with the notations used in the previous section, are

$$\begin{aligned} \varepsilon &= 10^{-3} \\ H_1 &= \begin{pmatrix} 10^{-4} & 0 & 0 \\ 0 & 10^{-1} & 0 \\ 0 & 0 & 10^{-4} \end{pmatrix} \\ H_2 &= \begin{pmatrix} 0.02 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.02 \end{pmatrix} \\ C &= -5\,10^{-4}\mathbf{I}_7 \end{aligned} \quad (47)$$

The initial non null conditions on the attitude error angles and their derivatives are due to the Minimum Impulse Bit of the 20 N thrusters used in the previous phase of operative attitude acquisition. In the most unfavourable situation a possible set of initial conditions is

$$\begin{aligned} \phi(0) &= 0.025 && \text{rad} \\ \theta(0) &= 0.025 && \text{rad} \\ \psi(0) &= 0.025 && \text{rad} \\ \dot{\phi}(0) &= 0.004 && \text{rad/s} \\ \dot{\theta}(0) &= 0.0012 && \text{rad/s} \\ \dot{\psi}(0) &= 0.0019 && \text{rad/s} \\ h(0) &= 0 && \text{N}\cdot\text{m}\cdot\text{s} \end{aligned} \quad (48)$$

In Figs. 3–4 the behaviour of the error angles and rates, respectively, are reported. In Fig. 5 the dipole momenta of the coils are presented and in Fig. 6 the wheel control voltage. It is assumed that the actuator bounds are $\mu_{\max} = 100\,\mathrm{As \cdot m^2}$ and $V_{\max} = 15\,\mathrm{V}$. In order to not exceed the bounds, the matrix C has been chosen to impose settling time of about 10^4 s. The simulations have been carried out taking into account environmental disturbances as atmospheric drag, solar pressure and gravity gradient. The figures evidence the robustness of the control law with respect to exogenous disturbances and model parameter uncertainties. Moreover noise–corrupted Earth magnetic field measures have been assumed. The insensitivity of the feedback law with respect to such measures lies on the *approximability property* (Utkin, 1978), (Bartolini and Zolezzi, 1986). This topic, due to space limitations, cannot be discussed in detail.

5 CONCLUSIONS

In this paper a sliding manifold approach to the attitude control of a satellite has been presented. In particular the proposed control law has been applied to the CARINA satellite. By means of a suitable definition of a *sliding manifold* for which the *equivalent control* is well defined we design a nonlinear controller which turns out to be the solution of a system of differential equations containing a small parameter. The approach is different from that of high gain systems because fast modes occur only in the control dynamics, and the control signal after a fast transient remains in a neighbourhood of the equivalent control. Then this technique retains the robustness properties of high gain systems with respect to external disturbances and plant parametric variations and avoids peaking phenomena. Simulations have been performed by means of the ESA–MIDAS dynamic simulator in presence of environmental disturbances and corrupted measures of the Earth magnetic field.

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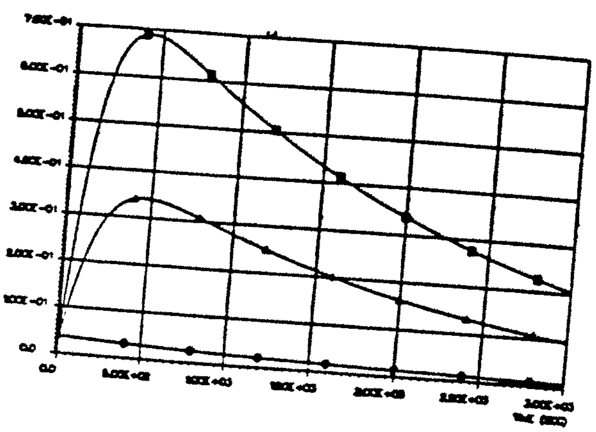

Figure 3: Euler error angles: □ = ϕ, ∘ = θ, △ = ψ

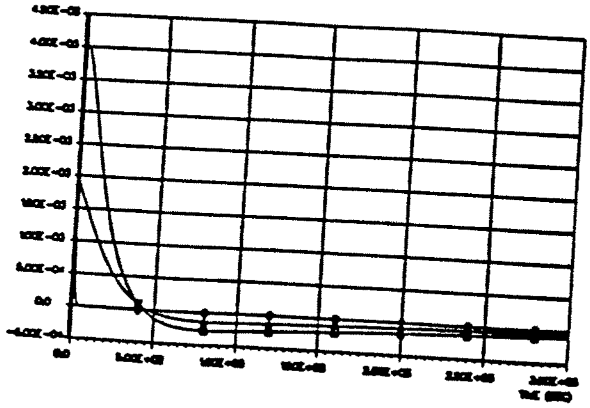

Figure 4: Euler error rates: $\square = \dot{\phi}$, $\circ = \dot{\theta}$, $\triangle = \dot{\psi}$

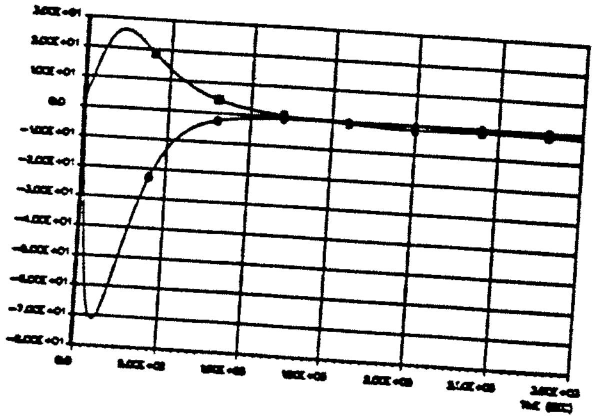

Figure 5: Dipole momenta of the coils: $\square = \mu_x$, $\circ = \mu_z$

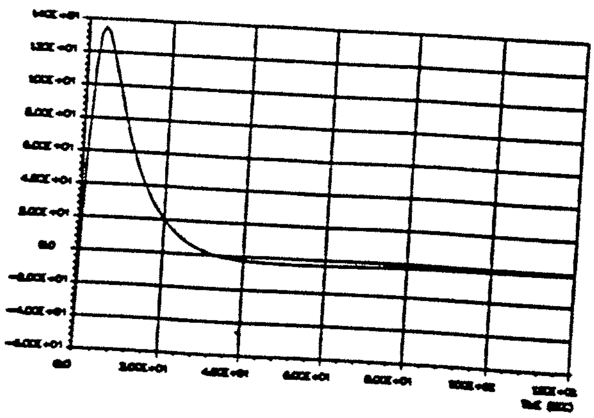

Figure 6: Wheel voltage