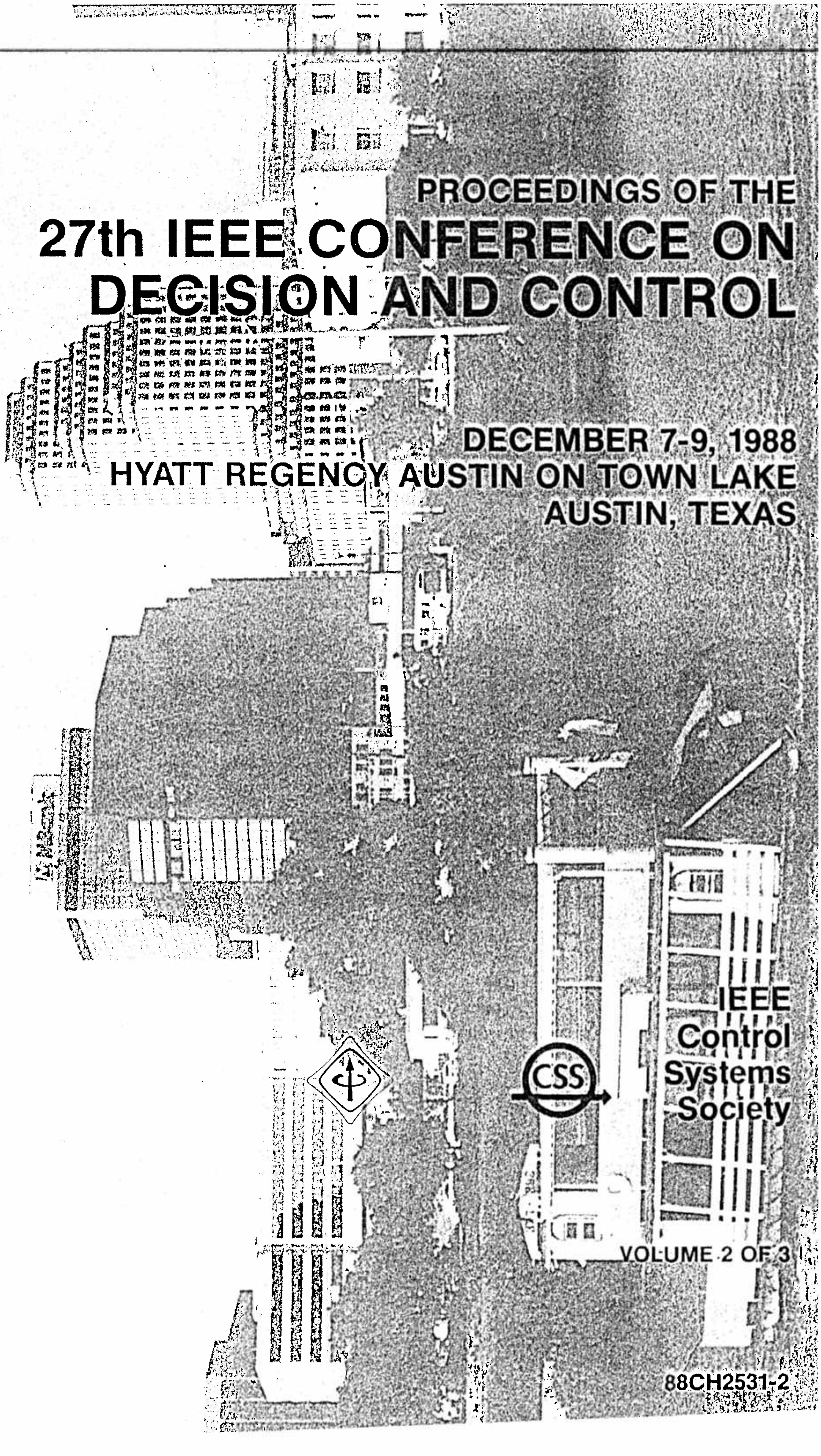

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ON THE LIMIT CYCLES IN FEEDBACK BILINEAR SYSTEMS

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ABSTRACT

In this note the presence of limit cycles in feedback bilinear systems is investigated. By using procedure quite similar to the sinusoidal describing function method an approximate solution is derived and then the existence and the uncertainty of an actual solution via techniques based on a continuation principle are stated.

I. INTRODUCTION

Consider a bilinear control system (BL) with unitary feedback (F) as described by the equations

$$\begin{cases} (\mathrm{BL}) \begin{cases} \dot{x} = Ax + uBx + bu \\ z = c^T x \end{cases} \\ (\mathrm{F}) \quad u = -z \end{cases} \qquad (\mathrm{BLF})$$

where $A, B \in \mathbb{R}^{n\times n}$ and $b, c \in \mathbb{R}^n$.
The functions $x:[0,+\infty)\to\mathbb{R}^n$, $u:[0,+\infty)\to\mathbb{R}$ and $z:[0,+\infty]\to\mathbb{R}$ respectively represent the state, the control and the output of the system and are infinitely derivable functions on $[0,+\infty)$.

The problem of the presence of limit cycles in the feedback bilinear system (BLF) and the study of their properties as magnitude, frequency and stability may be of relevant importance, for example, in the study of the behaviour in the large of such control systems (see [1]). The procedure followed to investigate the existence and the nature of oscillations is on the line of the well-known sinusoidal describing function method (see [2]). In the last twenty years several papers have provided theoretical justifications for such a method, showing that under suitable conditions the existence of an approximate solution implies the existence, whitin a certain distance, of a true periodic solution. ([3],[4], [5] and [6]).

The aim of this note is to give sufficient conditions based on the above mentioned approaches in order to guarantee the existence of a limit cycle $u=u(t)$, of some frequency $\omega>0$, of (BLF) or equivalently, according to our assumptions of the quadratic system in $\mathbb{R}^n$.

$$\dot{x} = (A - bc^T)x - c^T xBx \quad .$$

II. TOOLS AND PROCEDURE

The problem will be solved by means of conditions ensuring that when the first order harmonic balance equation associated to (BLF) admits a periodic solution, then there exists an actual periodic solution, in general of different period, of (BLF) "near" to the previous one.
In general the describing function method is applied to feedback systems which can be split in two scalar subsystems: a linear dynamic one defined by its transfer function and a nonlinear one (generally memoryless) represented by its describing function.
In the present case this approach is impossible since the nonlinear part given by the whole bilinear system (BL) does not allow an exact evaluation of its first harmonic response to a sinusoidal input. Therefore, an approximate describing function is proposed for representing the system (BL) such that the application of the classical describing function method for limit cycles corresponds, as usually happens, to the harmonic balance of the first order directly applied to the feedback configuration (BLF).
In order to characterize the bilinear system (BL) the following procedure is presented.
Consider the sinusoidal input $u_1(t) = \alpha\sin\omega t + \beta$, with $\alpha>0$ and $\omega>0$ and consider the dynamical system represented by the differential equation

$$\begin{cases} \dot{y}_1 = Ay_1 + BPy_1u_1 + bu_1 \\ z_1 = c^T y_1 \end{cases} \qquad (\mathrm{AS})$$

where $y_1(t) = k + p\sin\omega t + q\cos\omega t$, $t\in[0, 2\pi/\omega]$, $k,p,q\in\mathbb{R}^n$ and Pv represents the bias term and the first harmonics of the Fourier expansion of the function v.
Therefore (AS) approximates (BL) in the sense that (AS) is obtained from (BL), with input u_1, by means of the projection P and negleting the higher harmonics of the term y_1u_1.
It is easy to see that $z_1 = c^T y_1$ is a solution of (AS) if and only if k,p,q satisfy, for some $\omega>0$, the following system

$$\begin{cases} (A+\beta B)k + (\frac{1}{2}\alpha B)p = -\beta b \\ (A+\beta B)q - (\omega I)p = 0 \\ (\alpha B)k + (A+\beta B)p + (\omega I)q = -\alpha b \end{cases}$$

and

$$z_1 = c^T y_1$$

Let us denote by $k = k(\alpha,\beta,\omega)$, $p = p(\alpha,\beta,\omega)$ and $q = q(\alpha,\beta,\omega)$ the solution of such a system and by

$y_1 = y_1(u_1)$ the corresponding function.

The functions $\frac{c^T k}{\beta}$, $\frac{c^T(p+jq)}{\alpha}$ represent a kind of gain of the bilinear system (BL) for the bias and for the first harmonic component. In this sense they approximate the describing functions of the bilinear system (BL) and therefore the condition to have a limit cycle in the feedback system (BLF) becomes

$$\begin{cases} c^T \frac{k}{\beta} = -1 \\ c^T \frac{(p+jq)}{\alpha} = -1 \end{cases}$$

These equations are equivalent to

$$\begin{cases} c^T k + \beta = 0 \\ c^T p + \alpha = 0 \\ c^T q = 0 \end{cases}$$

or simply to

$$u_1 + c^T y_1(u_1) = 0 \qquad \text{(ASF)}$$

which represents the considered approximation of (BLF) and can also be derived as a direct first order harmonic balance on the same (BLF).

Finally, in order to connect (ASF) with (BLF) consider for $\lambda \in [0,1]$ the following family of systems (homotopy)

$$\begin{cases} u_1 = -c^T y_1(u_1) + \lambda[c^T y_1(u_1) - Pc^T x(u_1+u_2)] \\ u_2 = -\lambda(I-P)c^T x(u_1+u_2) \end{cases} \qquad (H)_\lambda$$

where $c^T x(u_1+u_2)$ represent the output of (BL) corresponding to the input $u = u_1+u_2 = Pu + (I-P)u$. It is evident that $(H)_0$ coincides with (ASF), while $(H)_1$ coincides with (BLF).

The next result gives the tool to deduce the existence of at least one non trivial (i.e. different from zero) isolated periodic solution of (BLF) from a non trivial isolated periodic solution of (ASF).

III. RESULT

We can prove the following

Theorem. (Continuation principle). Assume that there exists a connected bounded open set $\Omega_1 \subset \mathbb{R}^3$ with the following properties:

(i) $0 \notin \Omega_1$ and if $u = u_1+u_2$, with $u_1 = \alpha\sin\omega t + \beta$, is a non trivial $2\pi/\omega$-periodic solution of $(H)_\lambda$ for some $\lambda \in [0,1]$, then $(\alpha,\beta,\omega) \notin \partial\Omega_1$ and $\max_{t\in[0,2\pi/\omega]} |u_2(t)| < r(\alpha,\beta,\omega)$, where $r : \Omega_1 \to \mathbb{R}_+$ is a continuous function.

(ii) $\deg(I+c^T y_1(\cdot), J\Omega_1, 0) \neq 0$, where I is the identity and $J\Omega_1 = \{u_1 = \alpha\sin\omega t + \beta : (\alpha,\beta,\omega) \in \Omega_1\}$.
(deg (f,V,p) denotes the topological degree of the map f defined on the open set V with respect to the point p, see [7]).

Moreover, assume that

(iii) $\text{Re}\,\mu(A-bc)^T \le -\Lambda < 0$ and $\text{Re}\,\mu(A) \neq 0$, where $\text{Re}\,\mu(M)$ denotes the real part of the eigenvalues μ of the matrix M.

Then there exists a non trivial isolated periodic solution u of (BLF) with $u_1 = Pu \in J\Omega_1$.

The determination of the bounded connected open set Ω_1 and the proof of this result can be found in a more extensive paper of the authors [8].

III. EXAMPLE

Consider the single-input, single-output bilinear system

$$\begin{cases} \dot{x}_1 = -2x_2 + ux_2 - u \\ \dot{x}_2 = x_1 - 3x_2 - ux_1 + 1.1ux_2 - 2.9u \\ z = x_2 \end{cases}$$

which is connected by the unit feedback

$$u = -z$$

Fig. 1 shows the approximate solution y_1 derived by the proposed method, [8], i.e.

$$y_1 = \begin{pmatrix} 0.031 \\ -0.031 \end{pmatrix} + \begin{pmatrix} 0 \\ -0.245 \end{pmatrix} \sin 0.954t + \begin{pmatrix} -0.245 \\ 0 \end{pmatrix} \cos 0.954\,t$$

(hence $\bar{u}_1 = 0.031 + 0.245 \sin 0.954\,t$),
and the true isolated periodic solution $x = (x_1, x_2)$ obtained by numerical integration whose existence can also be proved by another approach as in [9].

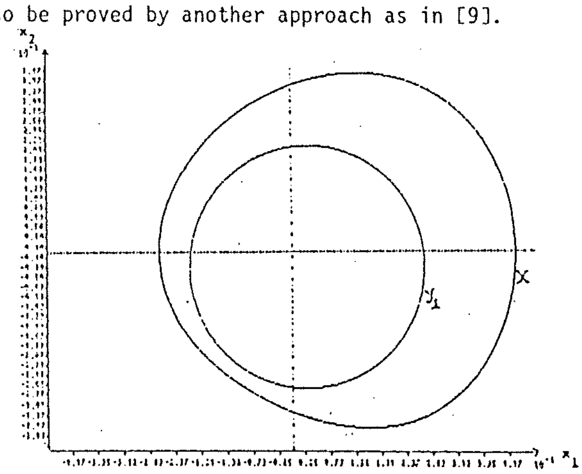

Fig. 1.

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