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CORRIGENDUM
NON-TRIVIAL NON-NEGATIVE PERIODIC SOLUTIONS OF A SYSTEM OF DOUBLY DEGENERATE PARABOLIC EQUATIONS WITH NONLOCAL TERMS

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Abstract. We correct a flaw in the proof of [1, Lemma 2.3].

1. **Corrigendum.** This Corrigendum concerns the proof of [1, Lemma 2.3]. In that proof there is a flaw in the estimate of $\log x_k$ due to an incorrect inequality. We provide here a correct estimate of $\log x_k$ in 1.5 which preserves the validity of Lemma 2.3. For the reader convenience we recall the statement of Lemma 2.3 and we give its complete proof. Here $m > 1$ and $p > 2$.

Lemma 2.3 *Let $K > 0$ and assume that u is a non-negative periodic function such that $u \in C(\overline{Q_T}), u^m \in L^p(0, T, W_0^{1,p}(\Omega))$ and satisfying*

$$u_t - \operatorname{div}\{[|\nabla(u^m + \epsilon u)|^2 + \eta]^{\frac{p-2}{2}} \nabla(u^m + \epsilon u)\} \le Ku, \quad \text{in } Q_T$$

and $u(\cdot, t)|_{\partial\Omega} = 0$, for $t \in [0, T]$. Then there exists $R > 0$ and independent of ϵ and η such that

$$\|u\|_{L^\infty} \le R.$$

Proof. We follow Moser's technique to show the stated a priori bounds. Multiplying

$$u_t - \operatorname{div}\{[|\nabla(u^m + \epsilon u)|^2 + \eta]^{\frac{p-2}{2}} \nabla(u^m + \epsilon u)\} \le Ku$$

by u^{s+1}, with $s \ge 0$, integrating over Ω and passing to the limit as $h \to 0$ in the Steklov averages u_h we have

$$\begin{aligned} K\|u(t)\|^{s+2}_{L^{s+2}(\Omega)} \ge &\frac{1}{s+2}\frac{d}{dt}\|u(t)\|^{s+2}_{L^{s+2}(\Omega)} \\ &+ (s+1)\int_\Omega [|\nabla(u^m + \epsilon u)|^2 + \eta]^{\frac{p-2}{2}} (mu^{m-1} + \epsilon)u^s|\nabla u|^2. \end{aligned}$$

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Since $p > 2$, $m > 1$ and

$$u^{(m-1)(p-2)}|\nabla u|^{p-2} \leq (mu^{m-1}+\epsilon)^{p-2}|\nabla u|^{p-2} \leq [|\nabla(u^m+\epsilon u)|^2+\eta]^{\frac{p-2}{2}},$$

we have

$$\frac{1}{s+2}\frac{d}{dt}\|u(t)\|^{s+2}_{L^{s+2}(\Omega)} + \int_\Omega u^{(p-1)(m-1)+s}|\nabla u|^p \leq K\|u(t)\|^{s+2}_{L^{s+2}(\Omega)}.$$

This implies

$$\begin{aligned} K(s+2)\|u(t)\|^{s+2}_{L^{s+2}(\Omega)} \geq& \frac{d}{dt}\|u(t)\|^{s+2}_{L^{s+2}(\Omega)} \\ &+ \frac{s+2}{[m(p-1)+s+1]^p}\int_\Omega \left|\nabla u^{\frac{m(p-1)+s+1}{p}}\right|^p. \end{aligned} \tag{1.1}$$

For ϵ and η fixed and $k = 1, 2, \ldots$, setting

$$s_k := 2p^k + \frac{p^k-p}{p-1} + m - 1, \quad \alpha_k := \frac{p(s_k+2)}{m(p-1)+s_k+1}, \quad w_k := u^{\frac{m(p-1)+s_k+1}{p}},$$

we obtain by 1.1

$$\frac{d}{dt}\|w_k(t)\|^{\alpha_k}_{L^{\alpha_k}(\Omega)} + \frac{s_k+2}{[m(p-1)+s_k+1]^p}\|\nabla w_k(t)\|^p_{L^p(\Omega)} \leq K(s_k+2)\|w_k(t)\|^{\alpha_k}_{L^{\alpha_k}(\Omega)}. \tag{1.2}$$

Observe that since $s_k \to +\infty$, as $k \to +\infty$, there exists k_0 such that $\alpha_k \in (1, p)$ for all $k \geq k_0$. By the interpolation and the Sobolev inequalities, it results

$$\|w_k(t)\|_{L^{\alpha_k}(\Omega)} \leq \|w_k(t)\|^{\theta_k}_{L^1(\Omega)}\|w_k(t)\|^{1-\theta_k}_{L^s(\Omega)} \leq C\|w_k(t)\|^{\theta_k}_{L^1(\Omega)}\|\nabla w_k(t)\|^{1-\theta_k}_{L^p(\Omega)}$$

for all $k \geq k_0$. Here $\theta_k = (s-\alpha_k)/[\alpha_k(s-1)]$, $s > p$ is fixed (say $s = p^*$ if $p < n$, where $p^* := np/(n-p)$) and C is a positive constant. Using the fact that $\|w_k(t)\|_{L^1(\Omega)} = \|w_{k-1}(t)\|^{\alpha_{k-1}}_{L^{\alpha_{k-1}}(\Omega)}$ and defining $x_k := \sup_{t\in\mathbb{R}}\|w_k(t)\|_{L^{\alpha_k}(\Omega)}$, one has

$$\begin{aligned} \|w_k(t)\|^{\frac{p}{1-\theta_k}}_{L^{\alpha_k}(\Omega)} \leq& C\|w_{k-1}(t)\|^{p\alpha_{k-1}\frac{\theta_k}{1-\theta_k}}_{L^{\alpha_{k-1}}(\Omega)}\|\nabla w_k(t)\|^p_{L^p(\Omega)} \\ \leq& C x_{k-1}^{p\alpha_{k-1}\frac{\theta_k}{1-\theta_k}}\|\nabla w_k(t)\|^p_{L^p(\Omega)}, \end{aligned}$$

for all $k \geq k_0$. Thus, by 1.2,

$$\begin{aligned} \frac{d}{dt}\|w_k(t)\|^{\alpha_k}_{L^{\alpha_k}(\Omega)} \leq& K(s_k+2)\|w_k(t)\|^{\alpha_k}_{L^{\alpha_k}(\Omega)} \\ &- C\frac{s_k+2}{[m(p-1)+s_k+1]^p}\|w_k(t)\|^{\frac{p}{1-\theta_k}}_{L^{\alpha_k}(\Omega)} x_{k-1}^{p\alpha_{k-1}\frac{\theta_k}{\theta_k-1}} \\ =& \left(K - \frac{C}{[m(p-1)+s_k+1]^p}\|w_k(t)\|^{\frac{p}{1-\theta_k}-\alpha_k}_{L^{\alpha_k}(\Omega)} x_{k-1}^{p\alpha_{k-1}\frac{\theta_k}{\theta_k-1}}\right) \\ &\cdot (s_k+2)\|w_k(t)\|^{\alpha_k}_{L^{\alpha_k}(\Omega)}, \end{aligned} \tag{1.3}$$

for all $k \geq k_0$. By Lemma 1.1 below, the differential inequality 1.3 implies

$$\|w_k(t)\|_{L^{\alpha_k}(\Omega)} \leq \left(\frac{K}{M_k}x_{k-1}^{p\alpha_{k-1}\frac{\theta_k}{1-\theta_k}}\right)^{\eta_k}, \tag{1.4}$$

for all $k \geq k_0$, where $\eta_k := (1-\theta_k)/[p-\alpha_k(1-\theta_k)]$ and $M_k := C/[m(p-1)+s_k+1]^p$. By definition of x_k and 1.4 we get

$$x_k \leq \left(\frac{K}{M_k}\right)^{\eta_k} x_{k-1}^{\nu_k}$$

for all $k \geq k_0$, with $\nu_k := p\alpha_{k-1}\theta_k/[p - \alpha_k(1-\theta_k)]$.

If $x_{k-1} \leq 1$, using the fact that $x_{k-1} = \sup_{t\in\mathbb{R}} \|u(t)\|_{s_{k-1}+2}^{\frac{m(p-1)+s_{k-1}+1}{p}}$, one has $\|u\|_{L^\infty} \leq 1$. Now, assume $x_{k-1} > 1$ and observe that there exists $\overline{k}_0$ such that, for all $k \geq \overline{k}_0$, $\eta_k \leq 1/(p\theta)$ and $\nu_k \leq p$. Here $\theta := (s-p)/[p(s-1)]$. Without loss of generality, assume $k_0 = \max\{\overline{k}_0, k_0\}$. Then, there exists a positive constant A such that

$$\begin{aligned} x_k \leq& \left(\frac{K}{C}\right)^{\eta_k} [m(p-1)+s_k+1]^{p\eta_k} x_{k-1}^{\nu_k} \\ \leq& \left(\frac{K}{C}\right)^{\eta_k} \left(mp + \frac{2p^{k+1}}{p-1}\right)^{p\eta_k} x_{k-1}^{\nu_k} \\ \leq& A p^{\frac{k+1}{\theta}} x_{k-1}^{p} \end{aligned}$$

for all $k \geq k_0$. Thus

$$\begin{aligned} \log x_k \leq& \log A + \frac{k+1}{\theta}\log p + p\log x_{k-1} \\ \leq& \log A \sum_{i=0}^{k-k_0-1} p^i + \frac{\log p}{\theta}\sum_{i=k_0+2}^{k+1} i p^{k+1-i} + p^{k-k_0}\log x_{k_0} \\ \leq& \frac{\log p}{\theta}\frac{p^{k-k_0}}{(p-1)^2}[k_0(p-1)+2p-1] \\ &+ \log A \frac{1-p^{k-k_0}}{1-p} + p^{k-k_0}\log x_{k_0}. \end{aligned} \tag{1.5}$$

Indeed, taking $x = \frac{1}{p}$ in $x\frac{d}{dx}\sum_{i=0}^{k+1} x^i = x\frac{d}{dx}\left(\frac{1-x^{k+2}}{1-x}\right)$, it results

$$\begin{aligned} \sum_{i=k_0+2}^{k+1} i p^{k+1-i} =& \frac{p^{k+3}}{(p-1)^2}\left[\frac{1}{p^{k+2}}\left(\frac{k+1}{p} - k - 2\right) - \frac{1}{p^{k_0+2}}\left(\frac{k_0+1}{p} - k_0 - 2\right)\right] \\ \leq& \frac{p^{k+3}}{(p-1)^2}\frac{1}{p^{k_0+2}}\left(k_0 + 2 - \frac{k_0+1}{p}\right) = \frac{p^{k-k_0}}{(p-1)^2}[k_0(p-1)+2p-1]. \end{aligned}$$

Then, by 1.5, it follows

$$x_k \leq A^{\frac{1-p^{k-k_0}}{1-p}} p^{\frac{p^{k-k_0}}{\theta(p-1)^2}[k_0(p-1)+2p-1]} x_{k_0}^{p^{k-k_0}}.$$

Since $x_k = \sup_{t\in\mathbb{R}} \|u(t)\|_{s_k+2}^{\frac{m(p-1)+s_k+1}{p}}$, we obtain

$$\begin{aligned} \|u(t)\|_{L^\infty(\Omega)} \leq& \lim_{k\to\infty} \|u(t)\|_{s_k+2} \\ \leq& \limsup_{k\to\infty}\left\{ A^{\frac{p}{m(p-1)+s_k+1}\frac{1-p^{k-k_0}}{1-p}} x_{k_0}^{\frac{p^{k-k_0+1}}{m(p-1)+s_k+1}} p^{\frac{p^{k-k_0+1}[k_0(p-1)+2p-1]}{\theta(p-1)^2(m(p-1)+s_k+1)}} \right\} \\ =&: R, \quad \forall\, t \in \mathbb{R}, \end{aligned}$$

where R is a positive constant. Hence $\sup_{t\in\mathbb{R}} \|u(t)\|_{L^\infty(\Omega)} \leq R$. It remains to prove that R is independent of ϵ and η as claimed. To this aim it is sufficient to prove that there exists $C > 0$ such that $x_{k_0} \leq C$. Indeed, by the inequality 1.1 with $s_0 := s_{k_0}$, it follows

$$\frac{d}{dt}\|u(t)\|^{s_0+2}_{L^{s_0+2}(\Omega)}+\frac{s_0+2}{[m(p-1)+s_0+1]^p}\int_\Omega\left|\nabla u^{\frac{m(p-1)+s_0+1}{p}}\right|^p\le K(s_0+2)\|u(t)\|^{s_0+2}_{L^{s_0+2}(\Omega)}. \tag{1.6}$$

Moreover, by the Hölder inequality with $r := \frac{m(p-1)+s_0+1}{s_0+2}$ and the Poincaré inequality, we have

$$\|u(t)\|^{m(p-1)+s_0+1}_{L^{s_0+2}(\Omega)} \le C\|\nabla u^{\frac{m(p-1)+s_0+1}{p}}\|^p_{L^p(\Omega)},$$

for a positive constant C. Thus, using 1.6, one has

$$\begin{aligned}
&\frac{d}{dt}\|u(t)\|^{s_0+2}_{L^{s_0+2}(\Omega)} + \frac{s_0+2}{C[m(p-1)+s_0+1]^p}\|u(t)\|^{m(p-1)+s_0+1}_{L^{s_0+2}(\Omega)}\\
&\le \frac{d}{dt}\|u(t)\|^{s_0+2}_{L^{s_0+2}(\Omega)} + \frac{s_0+2}{[m(p-1)+s_0+1]^p}\|\nabla u^{\frac{m(p-1)+s_0+1}{p}}\|^p_{L^p(\Omega)}\\
&\le K(s_0+2)\|u(t)\|^{s_0+2}_{L^{s_0+2}(\Omega)}.
\end{aligned}$$

Hence

$$\frac{d}{dt}\|u(t)\|^{s_0+2}_{L^{s_0+2}(\Omega)} \le \|u(t)\|^{s_0+2}_{L^{s_0+2}(\Omega)}\left(K(s_0+2) - M\|u(t)\|^{m(p-1)+s_0-1}_{L^{s_0+2}(\Omega)}\right),$$

where $M := \frac{s_0+2}{C[m(p-1)+s_0+1]^p}$. Lemma 1.1 implies

$$\|u(t)\|_{L^{s_0+2}(\Omega)} \le \{CK[m(p-1)+s_0+1]^p\}^{\frac{1}{m(p-1)+s_0-1}}, \quad \forall\, t\in\mathbb{R}.$$

Thus there exists $C>0$ such that $x_{k_0} = \sup_{t\in\mathbb{R}}\|u(t)\|_{s_0+2}^{\frac{m(p-1)+s_0+1}{p}} \le C$, as claimed. □

Lemma 1.1. *Let* $f : \mathbb{R} \to (0,+\infty)$ *be a differentiable and* T*-periodic function; suppose that there exist positive constants* s, α, β, γ *such that*

$$f'(t) \le f^s(t)(\beta - \gamma f^\alpha(t)),$$

for all $t\in\mathbb{R}$. *Then* $\beta-\gamma f^\alpha(t)\ge 0$ *for all* $t\in\mathbb{R}$.

We took advantage of this occasion to provide also an explicit estimate of x_{k_0} independent of ϵ and η, which shows that R is independent of these parameters.

We finally point out some misprints and imprecisions that could mislead the reader: at page 39, the uniqueness of the solution of (3) follows from [2, Theorem 32D], and Lemma 2.2 is proved by using [19, Theorem 1.2]; at the end of p. 40 the right equation for z is

$$l^{m,p}_{\epsilon,\eta,\sigma}[z] + Mz = 0$$

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