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CORRIGENDUM

MACKI J. W., NISTRI P. & ZECCA P., A tracking problem for uncertain vector systems, *Nonlinear Analysis* **14**(4), 319–328 (1990).

In the above paper the proof of theorem 3 is not correct. Specifically, as pointed out by G. Bartolini and T. Zolezzi, the error is in formula (7) p. 323, since we cannot put $\chi = \xi$ in (5) as z_δ depends on χ and ξ depends in turn on z_δ, in fact $\xi \in \partial V(s(t, y, z_\delta, C))$.

We want to give a correct (more restrictive) formulation of theorem 3.

The functions p and s, the sets $\mathcal{Q}$, U and all the formulas are as in [1].

Let m: $[0, \infty) \times \mathcal{Q} \times U \to \mathbb{R}$ be a function defined in the following way

$$m(t, x, u) = \inf_{\xi \in \partial V(s)} [\inf_{w \in F(t,x,u)} (\xi \cdot w - \xi \cdot p(t, y, v, C) - k^2)]$$

then it can be easily proved that the multivalued map $\hat{U}$: $[0, \infty) \times \mathcal{Q} -\circ\, U$, defined by

$$(t, x) -\circ\, \hat{U}(t, x) = \{u \in U: m(t, x, u) \geq 0\}$$

coincides with the map $\hat{U}$ defined in [1] and it is t-measurable, x-lower semicontinuous with nonempty closed values.

PROPOSITION 1. If we assume (H1), (H2) of [1] and the condition

(H3) $\forall\, (t, x) \in [0, \infty) \times \mathcal{Q}$ we have

$$F(t, x, \lambda u_1 + (1 - \lambda)u_2) \subseteq \lambda F(t, x, u_1) + (1 - \lambda)F(t, x, u_2), \qquad \forall\, u_1, u_2 \in U, \forall\, \lambda \in [0, 1],$$

then the set $\hat{U}(t, x)$ is convex.

Proof. Let $(t, x) \in [0, \infty) \times \mathcal{Q}$, $\forall\, \xi \in \partial V(s)$, with $s = s(t, y, x, C)$, we get

$$\inf_{w \in F(t,x,\lambda u_1 + (1-\lambda)u_2)} \xi \cdot w \geq \lambda \inf_{w \in F(t,x,u_1)} \xi \cdot w + (1 - \lambda) \inf_{w \in F(t,x,u_2)} \xi \cdot w \geq \xi \cdot p(t, y, v, C) + k^2$$

$\forall\, u_1, u_2 \in \hat{U}(t, x)$ and $\lambda \in [0, 1]$. Thus $\lambda u_1 + (1 - \lambda)u_2 \in \hat{U}(t, x)$. ∎

THEOREM 1. Let (y, v) be a fixed response-control pair for the linear model (2). Assume U is bounded, (H1), (H2) and (H3). Let x be any solution of (3) for any system dynamics f.

Then there exists $T \leq 2n((L + M + N)/k^2)$ such that

$$s(t) = y(t) - x(t) - e^{Ct}c_0 = 0 \qquad \text{for any } t \geq T.$$

Proof. Referring to the proof of theorem 3, as given in [1], we note only that since $(t, x) -\circ\, \hat{U}(t, x)$ is a t-measurable, x-lower semicontinuous map with nonempty, compact, convex values, Michael's continuous selection theorem establishes the existence of a selection $\bar{u}(t, x) \in \hat{U}(t, x)$ which is t-measurable and x-continuous.

Then the solutions of

$$\begin{cases} \dot{x} = f(t, x, \bar{u}(t, x)) \\ x(0) = x_0, \qquad |x_0| \le M \end{cases}$$

are a.e. solutions, i.e. solutions in the Caratheodory sense. Therefore

$$\frac{\mathrm{d}}{\mathrm{d}t} V(s(t)) = \xi \cdot (\dot{y}(t) - \dot{x}(t) - Ce^{Ct}c_0) = \xi \cdot p(t, y, v, C) - \xi \cdot \dot{x}(t) \le -k^2,$$

$$\forall\, \xi \in \partial V(s(t)). \quad \blacksquare$$

Let us observe that if condition (H2) is satisfied by the map $(t, x) \to H(t, x) = \bigcup_f G(t, x)$, where the union is taken on all the possible system dynamics f, then theorem 3 holds.

We remark that we have not been able to find either a correct proof or a counterexample for theorem 3 of [1].

REFERENCE

1. Macki J. W., Nistri P. & Zecca P., A tracking problem for uncertain vector systems, *Nonlinear Analysis* **14**, 319–328 (1990).