Test of Discrete Event Systems - 19.10.2020

A typical question concerning the event timing dynamics is what happens when the smallest residual lifetime corresponds to two or more events: how is the next event selected in such a situation? One possible way is to consider priorities between events. These priorities can be either set a priori, or are understood from the problem description. Exercise 1 deals with the latter case.

The basic event timing mechanism illustrated in the course relies on a certain number of assumptions. These assumptions imply that, when an event is not possible in the current state and becomes possible in the next state, a new lifetime is taken from the corresponding clock sequence. In other words, a new process underlying the event is started. This may not hold in some practical applications. An event may be paused and then resumed later when it becomes possible again. Exercise 2 shows an example of this type related to the aging process of a machine.

The basic event timing mechanism also assumes that, the next time an event becomes possible after its last occurrence, a new lifetime is taken from the corresponding clock sequence (and the corresponding score is increased by one). This may not hold in some practical applications. An example is round-robin queueing. Another example is given in Exercise 3, where the score of an event is increased according to different rules.

Exercise 1

A wireless sensor is powered by a battery of capacity 5 Ah. The sensor can be asked either to acquire a new measurement or to acquire a new measurement and transmit the measurement record via wireless to a data collector. In case of acquisition only, the battery discharge amounts to 1 Ah, whereas in case of both acquisition and transmission, it amounts to 2 Ah[1]. It is assumed that a request is accepted even if the remaining battery capacity is not sufficient to satisfy the request. The durations of both acquisition and transmission are assumed to be negligible. When the battery is too low, it is put on charge. During the charge, the wireless sensor is deactivated (meaning that all requests are rejected). The requests of acquisition arrive every 10 minutes, the requests of both acquisition and transmission arrive every 25 minutes, and the battery charge takes 18 minutes. The battery is initially full.

1. Compute the discharge time of the battery in steady state.

2. Compute the fractions of the two types of requests that are accepted in steady state.

[1] These values are not realistic.

Exercise 2

A machine can be in one of three states (idle, busy and down). The machine breaks down after 10 hours of operation (i.e. in state "busy"). When the machine breaks down, the ongoing job is lost. Repairing the machine takes 2 hours. After the repair, the machine is idle, waiting for a new job.

1. Define a logical model of the machine.

2. Assuming that: the machine is initially idle; the machine spends in state "idle" 3.0, 0.6, 1.0, 0.8 and 1.2 hours; the first five jobs require 1.8, 2.4, 2.2, 2.6 and 2.5 hours to be completed, determine how many jobs are completed before the machine breaks down for the first time.

Exercise 3

A doctor's office has a waiting room with only two chairs. Patients who arrive and find the waiting room full, do not have access to the doctor's office. Each patient has a maximum time he or she is willing to wait in the waiting room. If the patient is not received by the doctor within this time, the patient gives up and goes away.

1. Taking into account that: the doctor's office is empty at the opining; the first patient arrives after 2 minutes from the opening, and the others arrive after intervals of 1.5, 1.0, 2.0, 3.0, 3.5 minutes; the visit of the first patient requires 10 minutes; the maximum waiting times acceptable by the patients are 2.0, 6.5, 3.0, 4.0, 5.0, 3.5 minutes, compute how many patients give up during the visit of the first patient.

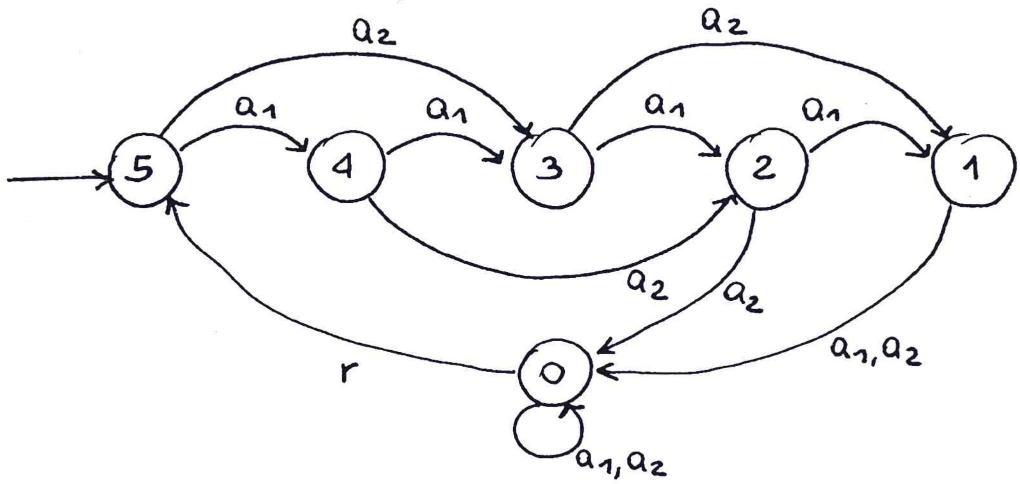
EXERCISE 1

model:

state x = battery level of charge $\in \{0,1,2,3,4,5\}$ ($x=0$ means that the battery is on charge)

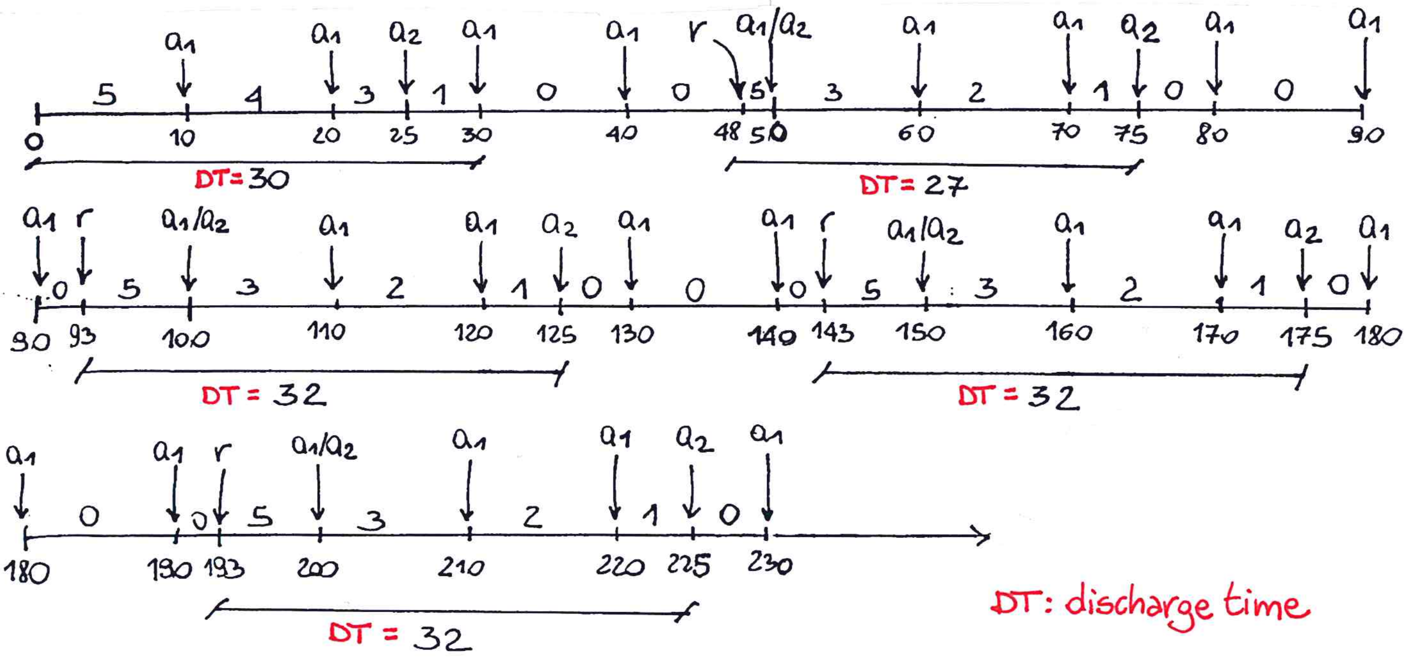
events $\mathcal{E} = \{a_1, a_2, r\}$

- a_1: request of acquisition only
- a_2: request of acquisition and transmission
- r: end of battery charge



TIPS:

- If a_1 and a_2 occur at the same time, it is reasonable to assume that a_2 has higher priority than a_1, since a_2 (acquisition and transmission) "includes" a_1 (only acquisition).
- For this system, steady state is meant when the discharge time does not change between cycles (a cycle begins with an event r and terminates with the next event r).



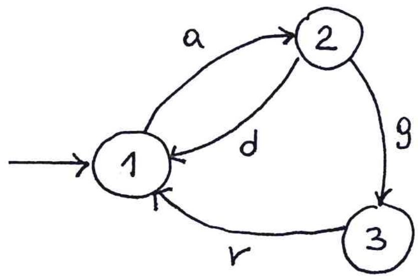
DT: discharge time

1. The discharge time of the battery at steady state is 32 min.
2. At steady state, all requests a_2 are accepted, while only 60% (fraction $\frac{3}{5}$) of requests a_1 are accepted (we consider that, when a_1 and a_2 occur simultaneously, a_1 is accepted because it is included in a_2).

EXERCISE 2

1) <u>model</u>:

state $x \in \{1, 2, 3\}$
- 1 → machine is IDLE
- 2 → machine is BUSY
- 3 → machine is DOWN



events $\mathcal{E} = \{a, d, g, r\}$
- a → start of a new job
- d → termination of a job
- g → breakdown of the machine
- r → end of repair

<u>Clock structure</u>:

$V = \{V_a, V_d, V_g, V_r\}$

$V_a = \{3.0, 0.6, 1.0, 0.8, 1.2\}$

$V_d = \{1.8, 2.4, 2.2, 2.6, 2.5\}$

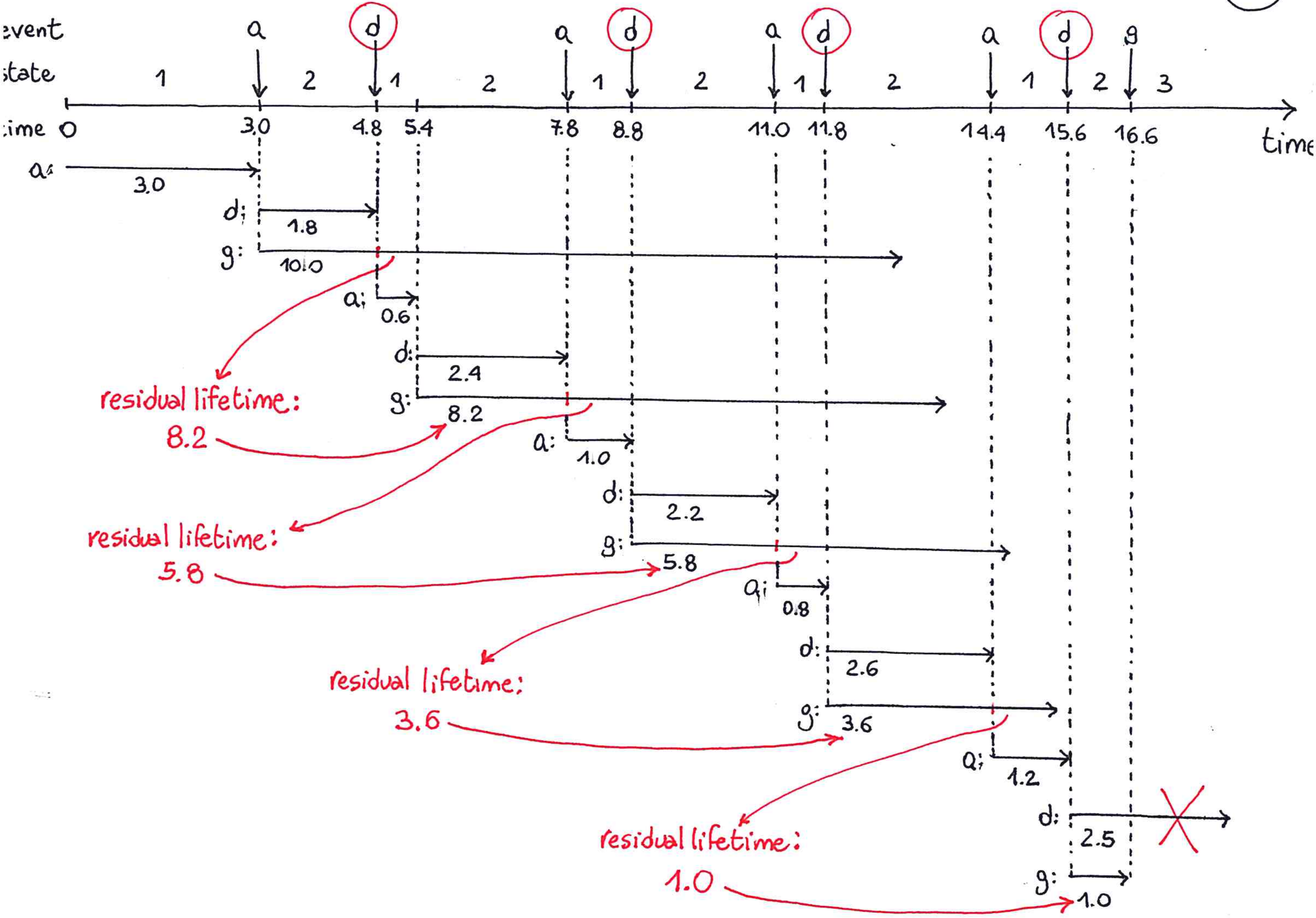
$V_g = \{10.0, \dots\}$

$V_r = \{2.0, \dots\}$

<u>REMARK</u>: Recall that, when event d occurs in state 2, event g is <u>paused</u> and is <u>resumed</u> when the machine returns busy.

<u>Sample path</u>:

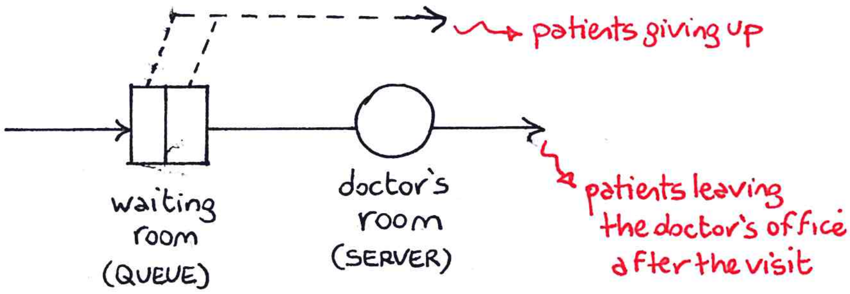
See the next page.



2) 4 jobs are completed before the first breakdown of the machine.

EXERCISE 3

The system can be represented as follows:

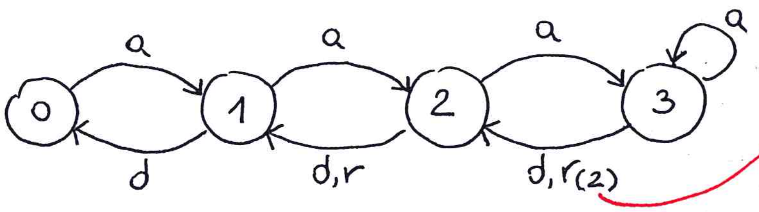


model:

x = # patients in the doctor's office $\in \{0,1,2,3\}$

events $\mathcal{E} = \{a, d, r\}$

- a: arrival of a patient
- d: termination of a visit
- r: patient gives up



This subscript means that there are two events r scheduled in parallel, because there are two patients in the waiting room, each one with his/her maximum waiting time

Let $i = 1,2,3,\ldots$ denote the patients' arrival order.

The subscript i will be used to highlight the patient an event refers to:

a_i: arrival of the i-th patient

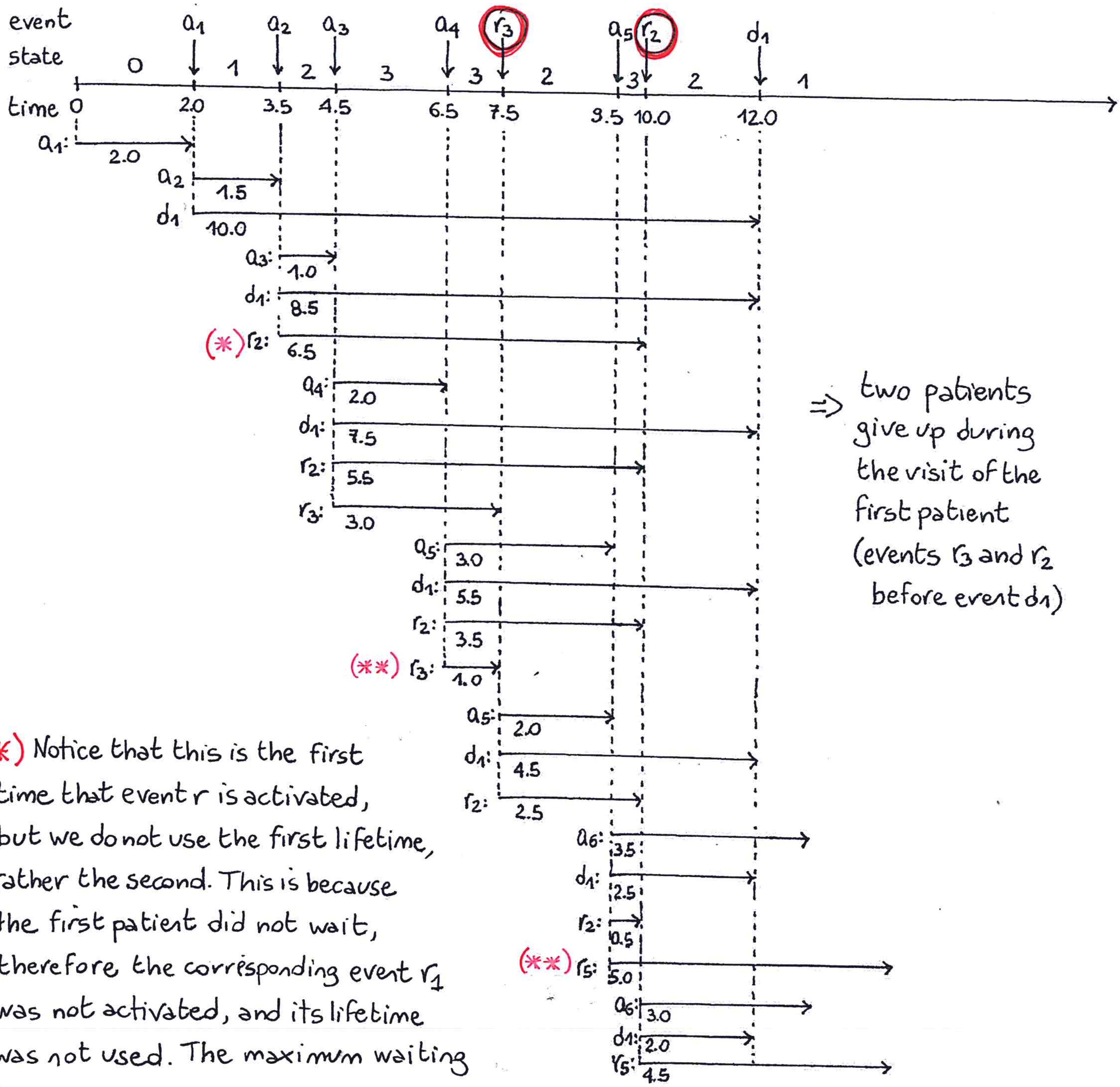
d_i: termination of the visit of the i-th patient

r_i: the i-th patient gives up

clock structure $V = \{V_a, V_d, V_r\}$

$V_a = \{\underset{a_1}{2.0}, \underset{a_2}{1.5}, \underset{a_3}{1.0}, \underset{a_4}{2.0}, \underset{a_5}{3.0}, \underset{a_6}{3.5}\}$, $V_r = \{\underset{r_1}{2.0}, \underset{r_2}{6.5}, \underset{r_3}{3.0}, \underset{r_4}{4.0}, \underset{r_5}{5.0}, \underset{r_6}{3.5}\}$, $V_d = \{\underset{d_1}{10.0}, \ldots\}$

Sample path:



⇒ two patients give up during the visit of the first patient (events r_3 and r_2 before event d_1)

(*) Notice that this is the first time that event r is activated, but we do not use the first lifetime, rather the second. This is because the first patient did not wait, therefore the corresponding event r_1 was not activated, and its lifetime was not used. The maximum waiting time is indeed a characteristic of the specific patient.

(**) The fourth patient is not accepted into the doctor's office, therefore the corresponding event r_4 is not activated, and its lifetime is not used. The fifth patient is accepted, and event r_5 is activated with the fifth lifetime in the clock sequence V_r.