

## Test of Discrete Event Systems - 18.12.2017

### Exercise 1

A small warehouse may contain up to three pallets. Every hour a truck comes to collect stored pallets. Depending on the space available on it, the maximum number of pallets that the truck may collect is 0 with probability  $p_0 = 0.1$ , 1 with probability  $p_1 = 0.2$ , 2 with probability  $p_2 = 0.4$  and 3 otherwise. The truck always collects as many pallets as possible compatibly with this constraint. The number of pallets shipped to the warehouse during one hour is a random variable taking values 0 with probability  $q_0 = 0.3$ , 1 with probability  $q_1 = 0.4$  and 2 otherwise. Pallets arriving when the warehouse is full are rejected. Assume that pallet loading and unloading times are negligible.

1. Model the system through a discrete-time homogeneous Markov chain.
2. In steady state condition, compute the average number of pallets in the warehouse after a truck departure.
3. Compute the probability that the number of pallets in the warehouse after a truck departure is two for exactly three consecutive times.

In all the subsequent questions, assume that the warehouse is initially empty.

4. Compute the probability that the number of pallets in the warehouse is never two after each of the first eight truck departures.
5. Compute the average number of hours to have the warehouse full after a truck departure.
6. Compute the probability to have the warehouse full after a truck departure before having it empty again.