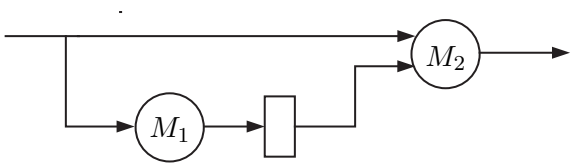
Test of Discrete Event Systems - 04.12.2017

Exercise 1

Consider the queueing network in the figure.



Arriving parts require preprocessing in M_1 with probability $p = 1/3$, otherwise they are routed directly to M_2. When a part arrives and the corresponding machine is unavailable, the part is rejected. There is a one-place buffer between M_1 and M_2. When M_1 terminates preprocessing of a part and M_2 is busy, the part is moved to the buffer, if the buffer is empty. Otherwise, the part is kept by M_1, that therefore remains unavailable for a new job until M_2 terminates the ongoing job. Parts arrive according to a Poisson process with expected interarrival time equal to 5 min, whereas service times in M_1 and M_2 follow exponential distributions with rates $\mu_1 = 0.5$ services/min and $\mu_2 = 0.8$ services/min, respectively.

1. Assume that the system is empty at time $t = 0$. Compute the probability that the system is full at time $t = 1$ h.
2. Compute the expected number of parts in the system at steady state.
3. Compute the expected time spent by a part in M_1 at steady state.
4. Verify the condition $\lambda_{eff} = \mu_{eff}$ for the whole system at steady state.
5. Compute the utilization of M_1 and M_2 at steady state.
6. Compute the blocking probability of the system at steady state for those parts requiring preprocessing in M_1.