Three exercises:

1) In Exercise 1 we apply the basic algorithm for the event timing dynamics, valid under assumptions A.1, A.2, and A.3.

2) In Exercise 2 event s is occasionally paused, and resumed later.

=> Assumption A.2 does not hold

⤳ To simulate the system correctly, we have to modify the basic algorithm: store the residual lifetime of the event, and use it later, when the event returns possible.

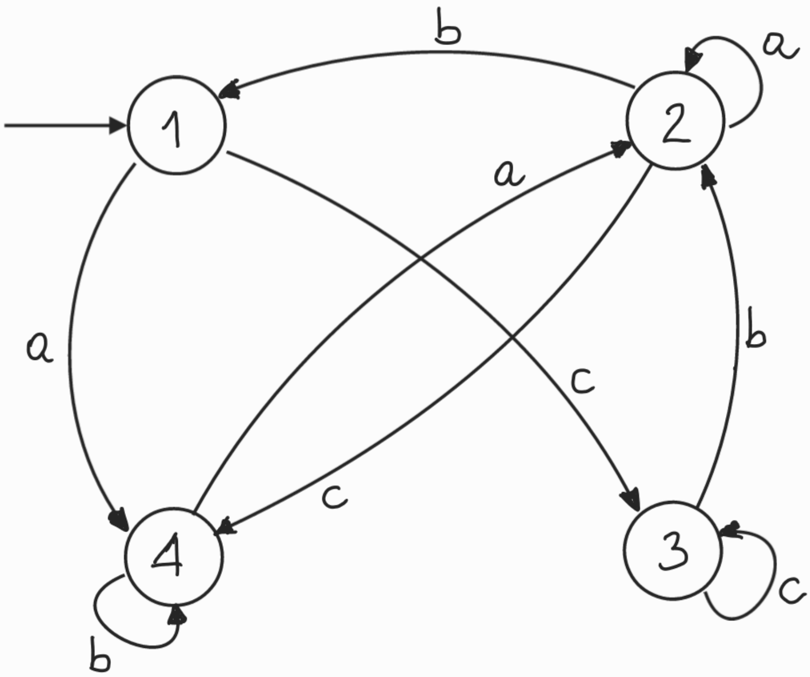
3) In Exercise 3 the score of event r may be increased even if the event is not activated.

↓ This rule requires a modification of the basic algorithm, being a case not considered in step 4.

EXERCISE 1

Consider the timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0, V)$ with:

- $\mathcal{X} = \{1, 2, 3, 4\}$
- $\mathcal{E} = \{a, b, c\}$
- Γ, f and x_0 defined by the following graph:



- the clock structure $V = \{V_a, V_b, V_c\}$ is composed of the clock sequences:

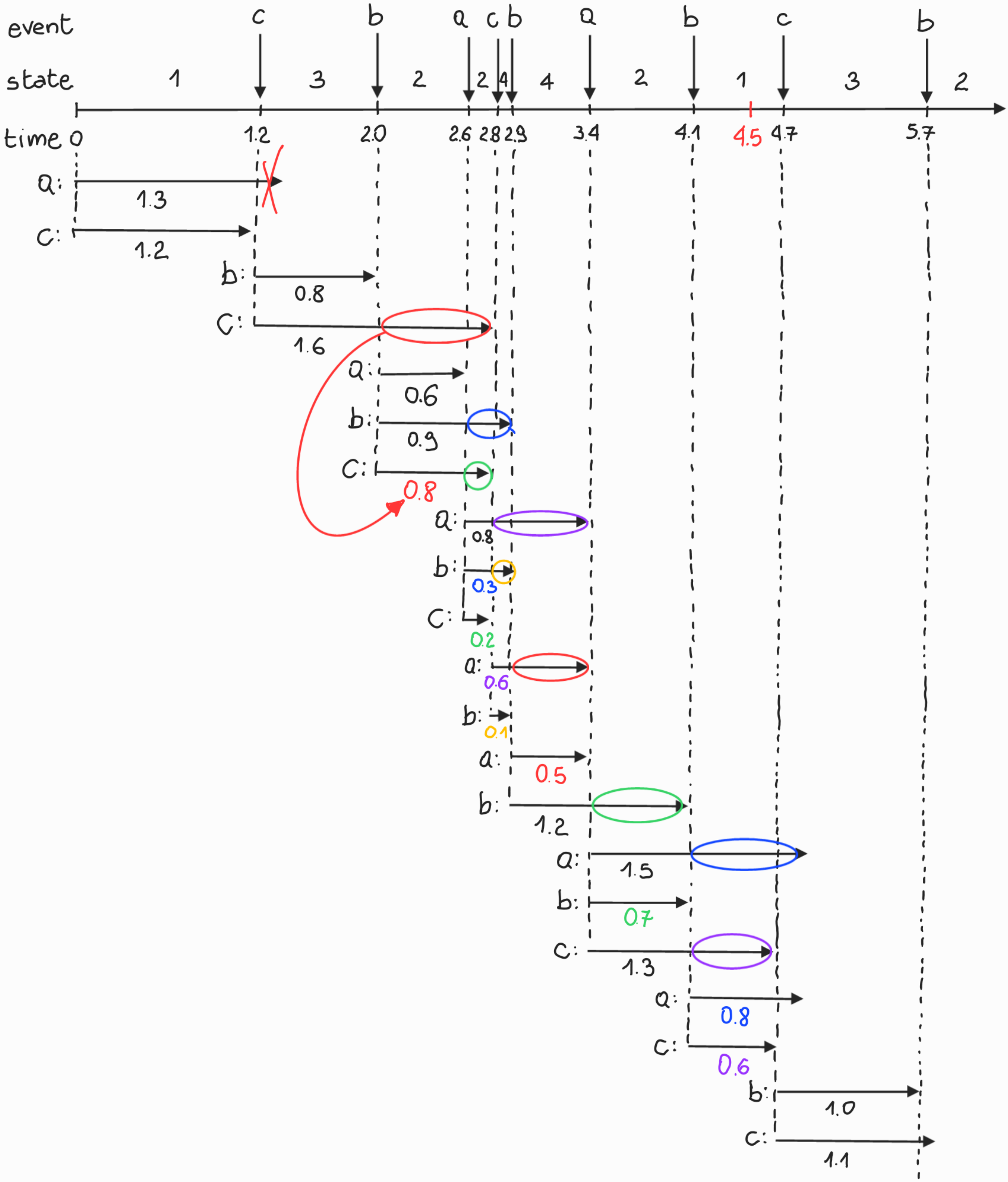
$V_a = \{1.3, 0.6, 0.8, 1.5\}$

$V_b = \{0.8, 0.9, 1.2, 1.0\}$

$V_c = \{1.2, 1.6, 1.3, 1.1\}$

Assume that assumptions A.1, A.2 and A.3 hold.

1) Determine the sample path of the system.



2) Determine the state at time $t = 4.5$

From the sample path: $x(4.5) = 1$

3) Compute the fraction of time spent in each state over the interval $[0, 4.5]$.

time spent in state 1

$$f_1 = \frac{(1.2-0)+(4.5-4.1)}{4.5} = \frac{1.6}{4.5} \simeq 35.56\,\%$$

time horizon

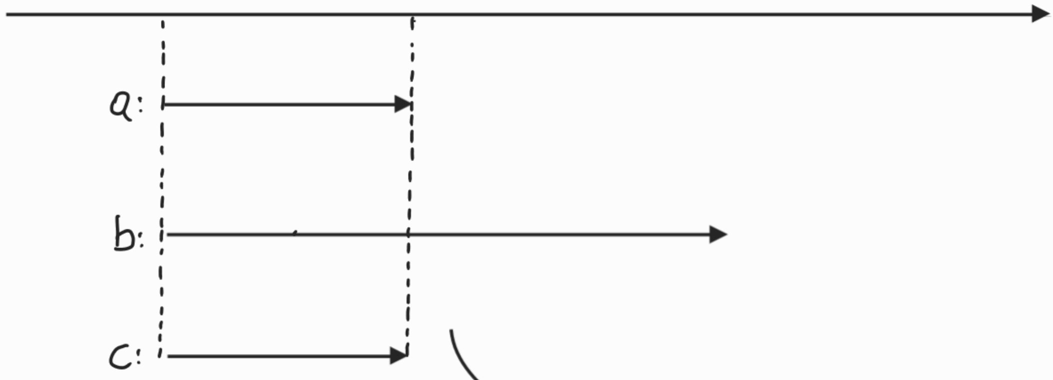
$$f_2 = \frac{(2.6-2)+(2.8-2.6)+(4.1-3.4)}{4.5} = \frac{1.5}{4.5} \simeq 33.33\,\%$$

$$f_3 = \frac{(2.0-1.2)}{4.5} = \frac{0.8}{4.5} \simeq 17.78\,\%$$

$$f_4 = \frac{(2.9-2.8)+(3.4-2.9)}{4.5} = \frac{0.6}{4.5} \simeq 13.33\,\%$$

REMARK

What happens when two (or more) events have the smallest residual lifetime?



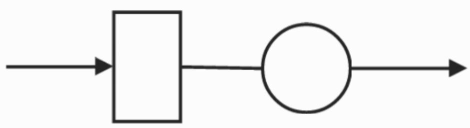
Which event occurs next: a or c?

$\Rightarrow$ Use priorities between events

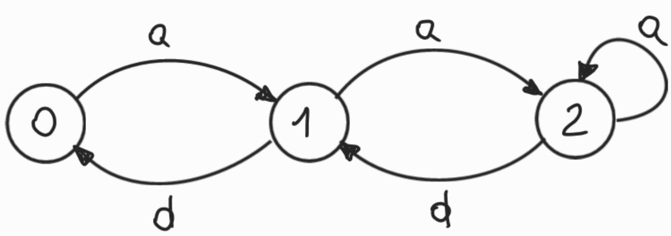
In real-world problems, these priorities must be understood from the context.

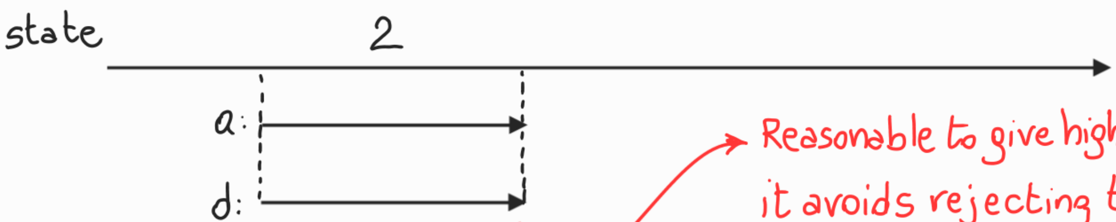
Example: queueing system

$\mathcal{E} = \{a, d\}$



$x = \#\text{customers} \in \{0, 1, 2\}$





Reasonable to give higher priority to d: it avoids rejecting the arriving customer

EXERCISE 2

A machine is *idle*, when it waits for the arrival of a raw part, and *busy*, when it processes a part. Arrivals of raw parts are suspended when the machine is in state *busy*. The machine is subject to ageing only when it is in state *busy*. Ageing is modelled by a deterministic function of the type $u = 0.01t_b + 0.009t_b^2$, where t_b [min] is the total time spent in state *busy* from the end of the last maintenance. The machine is stopped for maintenance when $u = 1$. The duration of maintenance is 5.0 min. After maintenance, the machine resumes the job that was previously stopped.

Assume that the machine is initially in state *idle* with $u = 0$. Time intervals spent in state *idle* have durations of 1.4, 0.6, 0.4, 1.2, 0.8, 1.8, and 1.5 min. Processing of the first seven raw parts requires 1.8, 1.6, 1.5, 2.4, 2.1, 1.5, and 2.0 min, respectively.

1. Determine the sample path of the system.
2. Compute the utilization and the throughput of the machine from the initial time to the start of the first maintenance.

1. First, we determine the time spent in state busy when maintenance is started.

$$\begin{cases} u = 0.01t_b + 0.009t_b^2 \\ u = 1 \end{cases} \Rightarrow \boxed{0.009t_b^2 + 0.01t_b - 1 = 0}$$

quadratic equation

$$\Delta = (0.01)^2 + 4 \cdot 0.009 = 0.0361$$

$$\sqrt{\Delta} = 0.19$$

$$t_b = \frac{-0.01 \pm 0.19}{0.018} = \begin{cases} 10 \\ \cancel{-\frac{100}{9}} \end{cases}$$

we accept only positive solutions

Then, we model the system with a timed automaton

$(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0, V)$

events $\mathcal{E} = \{a, d, s, m\}$

a: arrival of a raw part

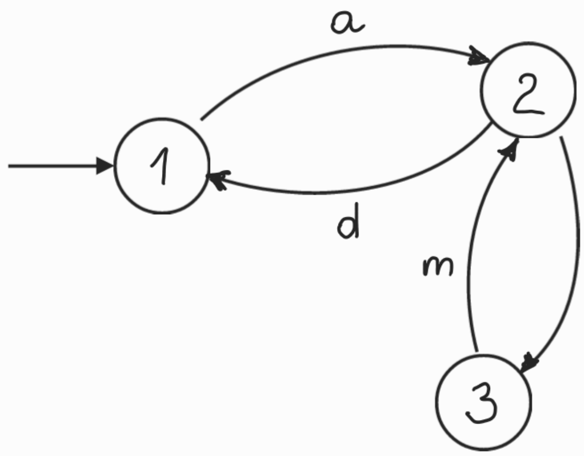
d: termination of processing

s: start of maintenance

m: termination of maintenance

state $x = \begin{cases} 1 : \text{machine is idle} \\ 2 : \text{machine is busy (working)} \\ 3 : \text{machine is under maintenance} \end{cases}$

state transition diagram



clock structure: $V = \{V_a, V_d, V_s, V_m\}$

$V_a = \{1.4, 0.6, 0.4, 1.2, 0.8, 1.8, 1.5\}$

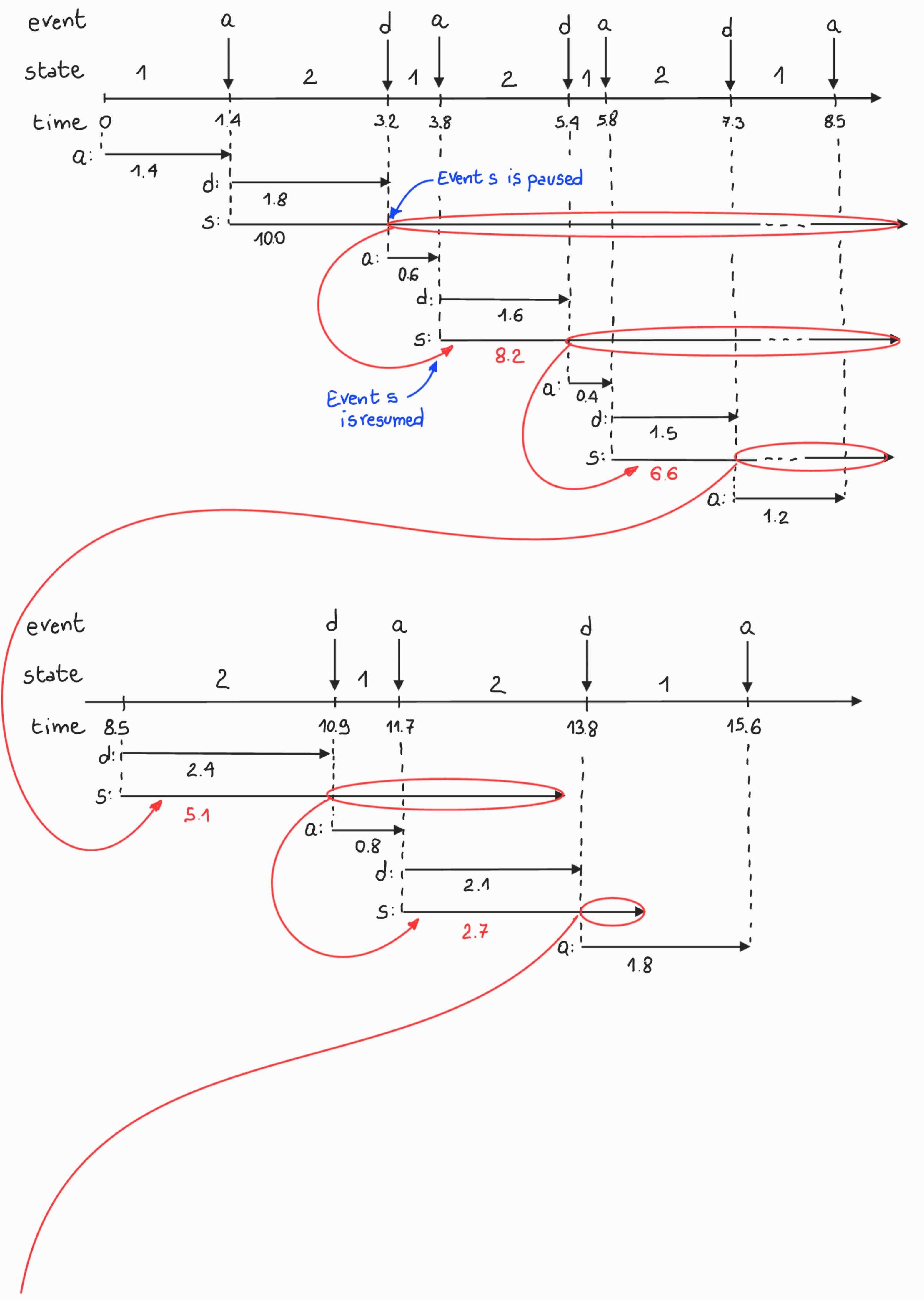
Since only event a is possible in state 1, and when a occurs, the state changes to 2, the lifetimes of event a coincide with the time intervals spent in state 1.

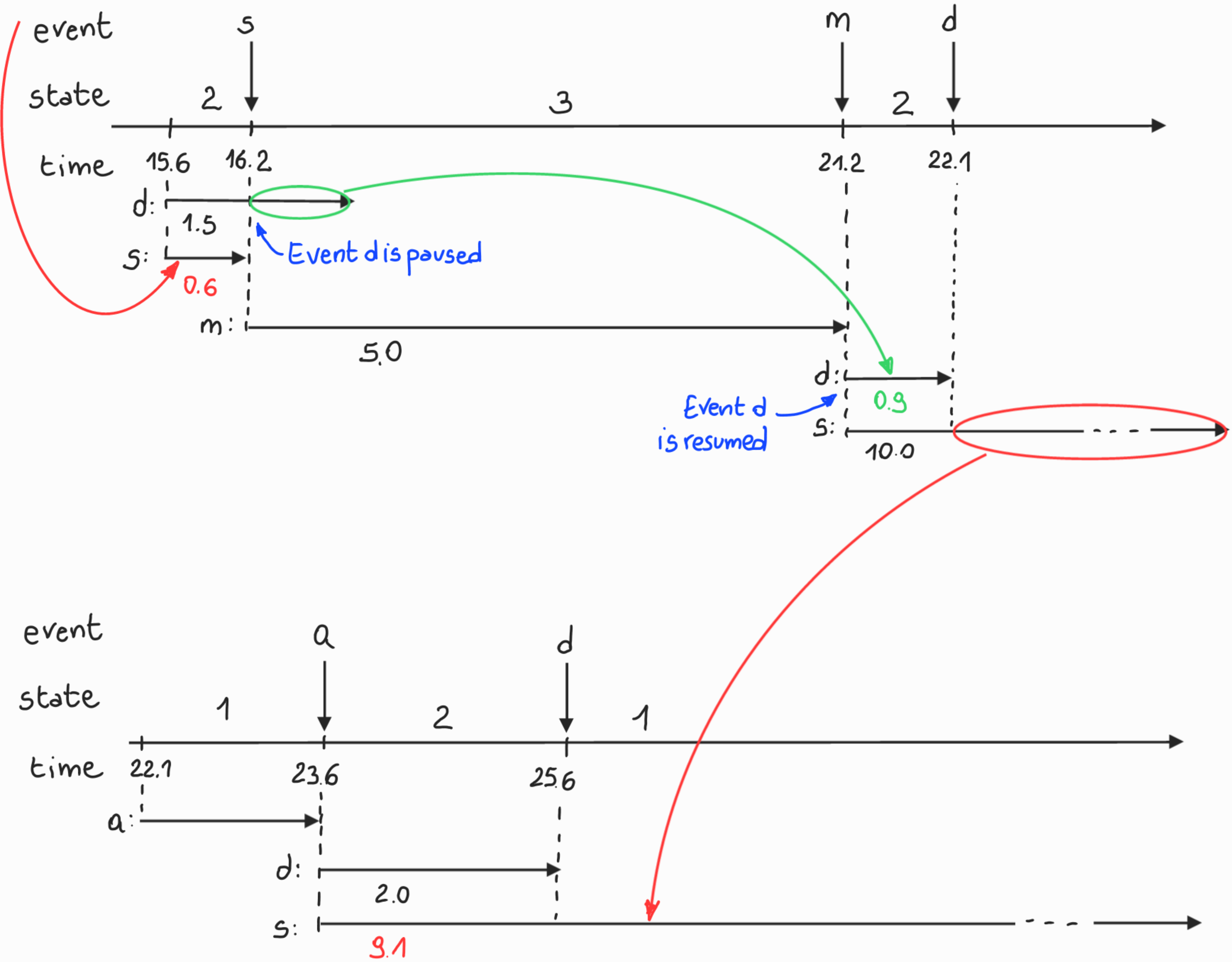
$V_d = \{1.8, 1.6, 1.5, 2.4, 2.1, 1.5, 2.0\}$

$V_s = \{10.0, 10.0, \ldots\}$

$V_m = \{5.0, \ldots\}$

This is the value of t_b computed at the beginning of the exercise


event
state
time
a
d
1
2
0
1.4
3.2
3.8
5.4
5.8
7.3
8.5
a:
1.4
d:
1.8
s:
10.0
Event s is paused
0.6
1.6
8.2
Event s is resumed
0.4
1.5
6.6
1.2
10.9
11.7
13.8
15.6
2.4
5.1
0.8
2.1
2.7
1.8



Notice that events d and s do not satisfy A.2, because if they are possible in the current state, they do not occur, and they are not possible in the next state, then they are paused. Hence, the previous algorithm could not be applied as it is, but we had to modify it in order to simulate correctly the system.

2. The first maintenance starts at time $t = 16.2$.

The utilization U of a machine is the fraction of time that the machine is working.

Intervals when the machine is working up to time $t = 16.2$

$$U = \frac{(3.2-1.4)+(5.4-3.8)+(7.3-5.8)+(10.9-8.5)+(13.8-11.7)+(16.2-15.6)}{16.2}$$

Time horizon

$$= \frac{10.0}{16.2} \simeq 61{,}73\ \%$$

The throughput μ of a machine is the average number of outputs (products) per time unit.

Number of events d up to time $t = 16.2$

$$\mu = \frac{5}{16.2} \simeq 0.31 \text{ products/min}$$

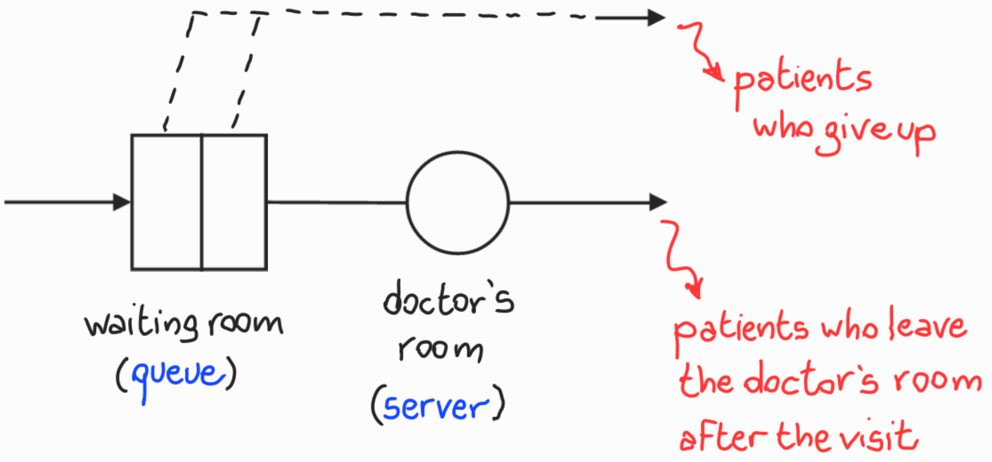
Time horizon

Exercise 3

A doctor's office has a waiting room with only two chairs. Patients who arrive and find the waiting room full, do not have access to the doctor's office. Each patient has a maximum time he or she is willing to wait in the waiting room. If the patient is not received by the doctor within this time, the patient gives up and goes away.

1. Taking into account that: the doctor's office is empty at the opining; the first patient arrives after 2 minutes from the opening, and the others arrive after intervals of 1.5, 1.0, 2.0, 3.0, 3.5 minutes; the visit of the first patient requires 10 minutes; the maximum waiting times acceptable by the patients are 2.0, 6.5, 3.0, 4.0, 5.0, 3.5 minutes, compute how many patients give up during the visit of the first patient.

The system can be represented as follows:



model

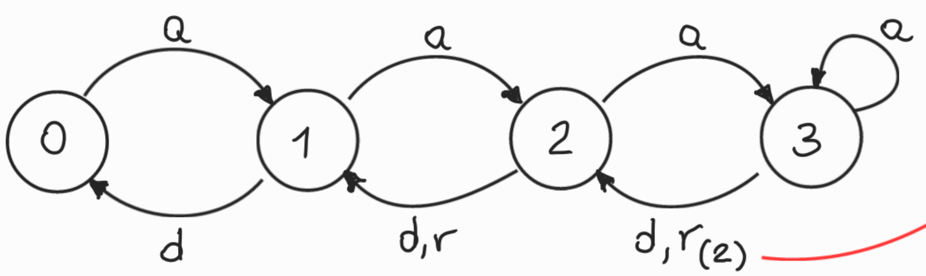
x = # patients in the doctor's office $\in \{0, 1, 2, 3\}$

events $\mathcal{E} = \{a, d, r\}$

a: arrival of a patient

d: termination of a visit

r: a patient gives up



This subscript means that there are two events r scheduled in parallel, because there are two patients in the waiting room, each one with his/her maximum waiting time

Since the maximum waiting time is a "property" of the specific patient, we distinguish the patients with their order of arrival.

Let $i = 1, 2, 3, \ldots$ denote the patient's order of arrival.

The subscript i is used to identify the patient an event refers to:

- a_i : arrival of the i-th patient
- d_i : termination of the visit of the i-th patient
- r_i : the i-th patient gives up

clock structure: $V = \{V_a, V_d, V_r\}$

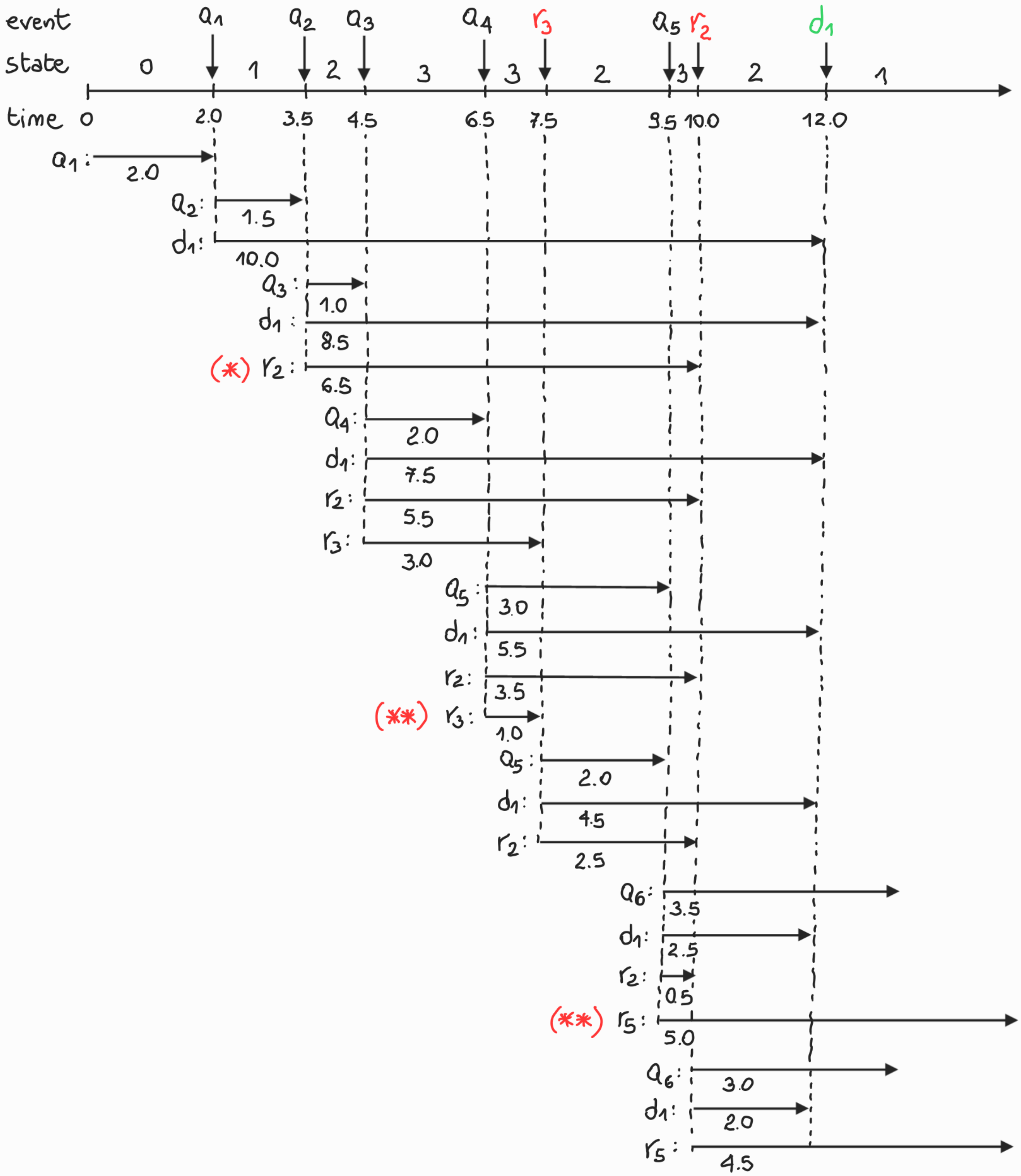
$$V_a = \{\underset{a_1}{2.0}, \underset{a_2}{1.5}, \underset{a_3}{1.0}, \underset{a_4}{2.0}, \underset{a_5}{3.0}, \underset{a_6}{3.5}\}, \quad V_d = \{\underset{d_1}{10.0}, \ldots\}, \quad V_r = \{\underset{r_1}{2.0}, \underset{r_2}{6.5}, \underset{r_3}{3.0}, \underset{r_4}{4.0}, \underset{r_5}{5.0}, \underset{r_6}{3.5}\}$$

sample path: see the next page

REMARKS

(*) Notice that this is the first time that event r is activated, but we use the second lifetime, not the first one. This is because the first patient did not wait, thus the corresponding event r_1 was not activated, and its lifetime was not used. Recall that the maximum waiting time is a property of the specific patient. Here the second patient is waiting, thus we activate his/her event r_2.

(**) The fourth patient is not accepted in the doctor's office, thus the corresponding event r_4 is not activated, and its lifetime is not used. The fifth patient is accepted instead, and event r_5 is activated with the fifth lifetime in the clock sequence V_r.



⟹ Two patients give up during the visit of the first patient (events r_3 and r_2 before event d_1).