Three exercises:

1) scheduling of operations on a machine

2) series connection of queueing systems

3) queueing system with parallel servers

EXERCISE 1

A machine performs operations of three types, denoted a, b and c. For technical reasons, an operation of type c cannot be performed immediately after two consecutive operations both of type a, or both of type b. At initialization, no operation has been performed.

1. Model the logic of the machine.
2. Model a system designed to support the scheduling of the operations on the machine: given a sequence of operations, the system returns whether the sequence is feasible for the machine, or not.

1. We model the system logic with a state automaton

$(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0)$

events $\mathcal{E} = \{a, b, c\}$

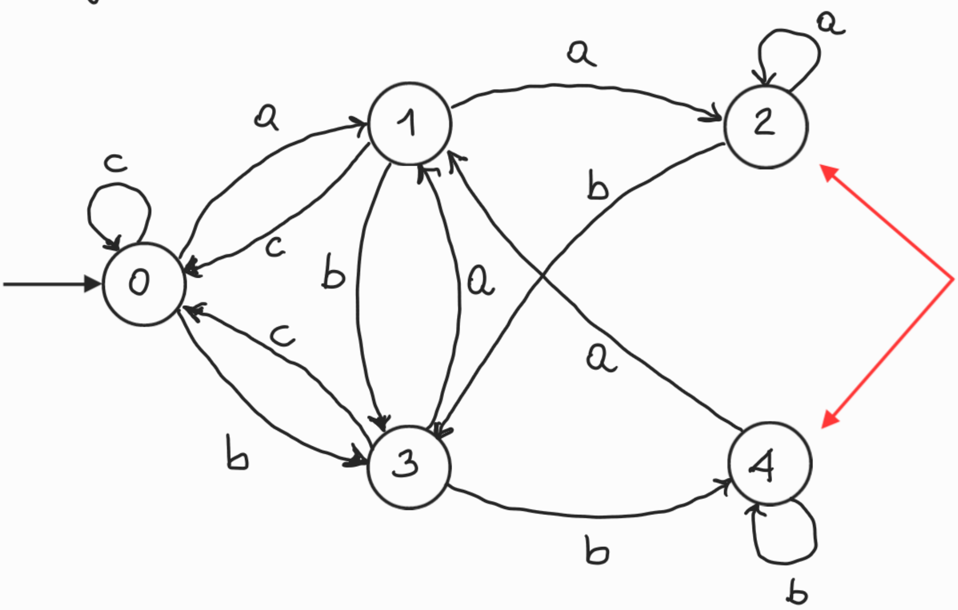
- a → performed operation a
- b → performed operation b
- c → performed operation c

The definition of state for this system should take into account the last two operations performed

→ information needed to figure out whether operation c can be performed next or not

$$x = \begin{cases} 1: & \text{last operation } a, \text{ second last } \underline{\text{not}}\ a \\ 2: & \text{last two operations } a \\ 3: & \text{last operation } b, \text{ second last } \underline{\text{not}}\ b \\ 4: & \text{last two operations } b \\ 0: & \text{otherwise} \end{cases}$$

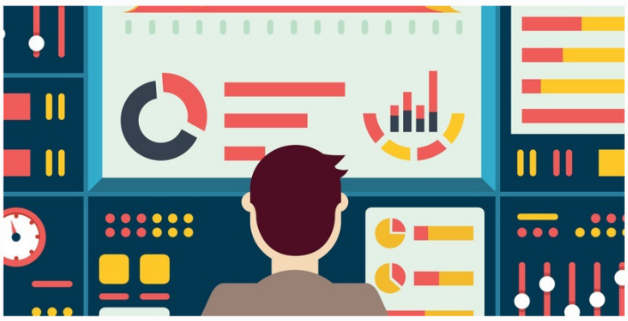
Γ, f and x_0 are defined by the following graph:



According to the technical constraint, event c is <u>not possible</u> in states 2 and 4

This model is useful, e.g., for <u>monitoring</u>:

a monitoring software receives information from the machine (operation performed) and updates the estimated state of the machine, which is visualized on the screen of an operator.



$\rightsquigarrow$ The sequence of operations received from the "field" is always <u>feasible</u> (except for possible communication errors...)

2. We modify the previous model:

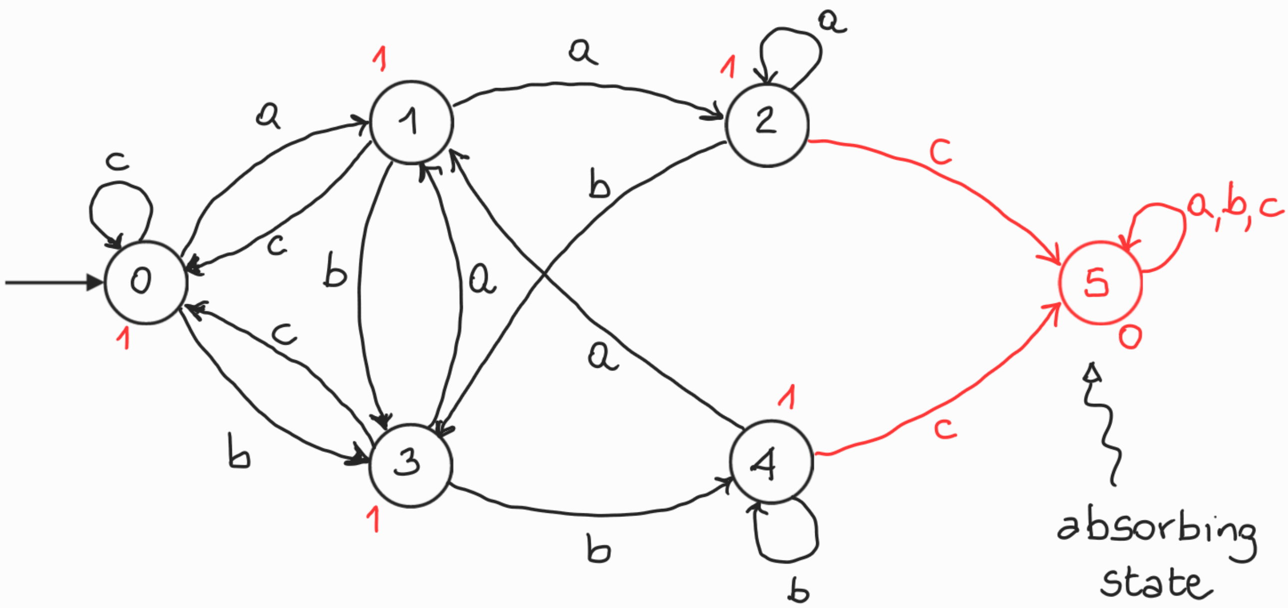
- we add a dummy state collecting error situations

  state 5: error

- we define the following output:

$$y = \begin{cases} 0 & \text{if the sequence of operations is infeasible} \\ 1 & \text{otherwise} \end{cases}$$

Resulting model:



This model is useful, e.g., for planning:
given a sequence of operations generated in a planning process, if the final output is 0, the sequence is infeasible, and feasible otherwise.

example: determine whether the sequence of operations

caabcbbcab

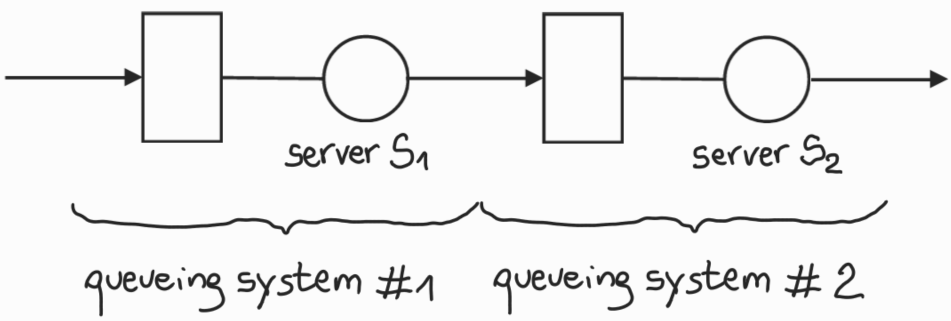
is feasible for the machine

event		c	a	a	b	c	b	b	c	a	b
state	0	0	1	2	3	0	3	4	5	5	5
output	1	1	1	1	1	1	1	1	0	0	0

the final output is 0: the sequence is infeasible

EXERCISE 2

Series connection of two queueing systems:



Technical constraint: if the second queueing system is full when S_1 terminates a service, S_1 keeps the customer, and is not available for a new service until the customer can move forward (the server is blocked).

We model the system with a state automaton $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0)$:

events $\mathcal{E} = \{a, d_1, d_2\}$

- a → arrival of a customer
- d_1 → termination of a service in S_1
- d_2 → termination of a service in S_2

state $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where $x_1 \rightarrow$ queueing system #1: $x_1 \in \{0, 1, 2, 3, 4\}$ and $x_2 \rightarrow$ queueing system #2: $x_2 \in \{0, 1, 2\}$

where:

0 : empty

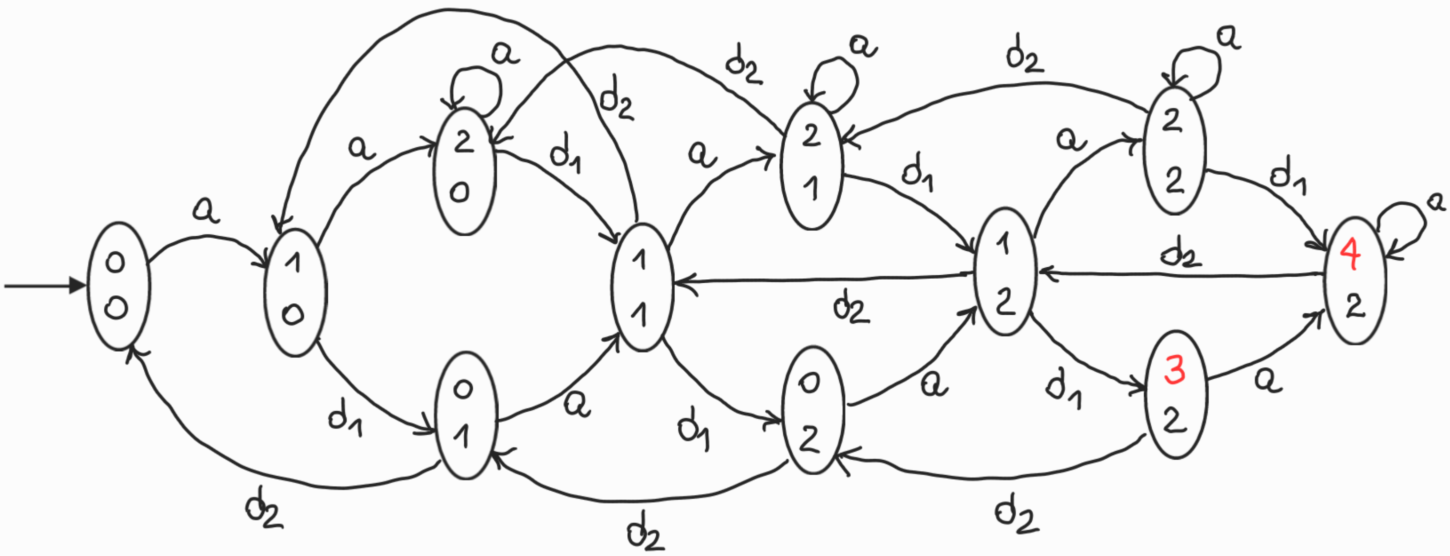
1 : one customer, server working

2 : two customers, server working

3 : one customer, server blocked

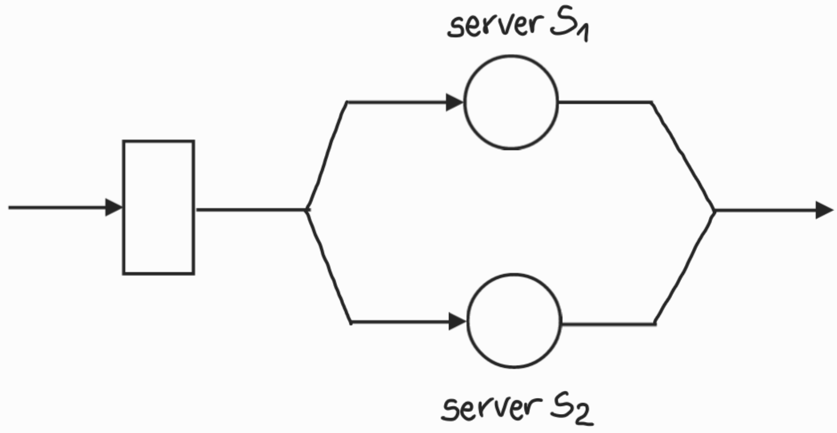
4 : two customers, server blocked

Γ, f and x_0 are defined by the following graph (we assume that the system is initially empty):



EXERCISE 3

Queueing system with parallel servers:



The two servers perform the same service, but server S_1 is faster than server S_2 $\Rightarrow$ the two servers are different

Rule for routing the customers: if both servers are available, the next arriving customer is served by S_1

We model the system with a state automaton $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0)$:

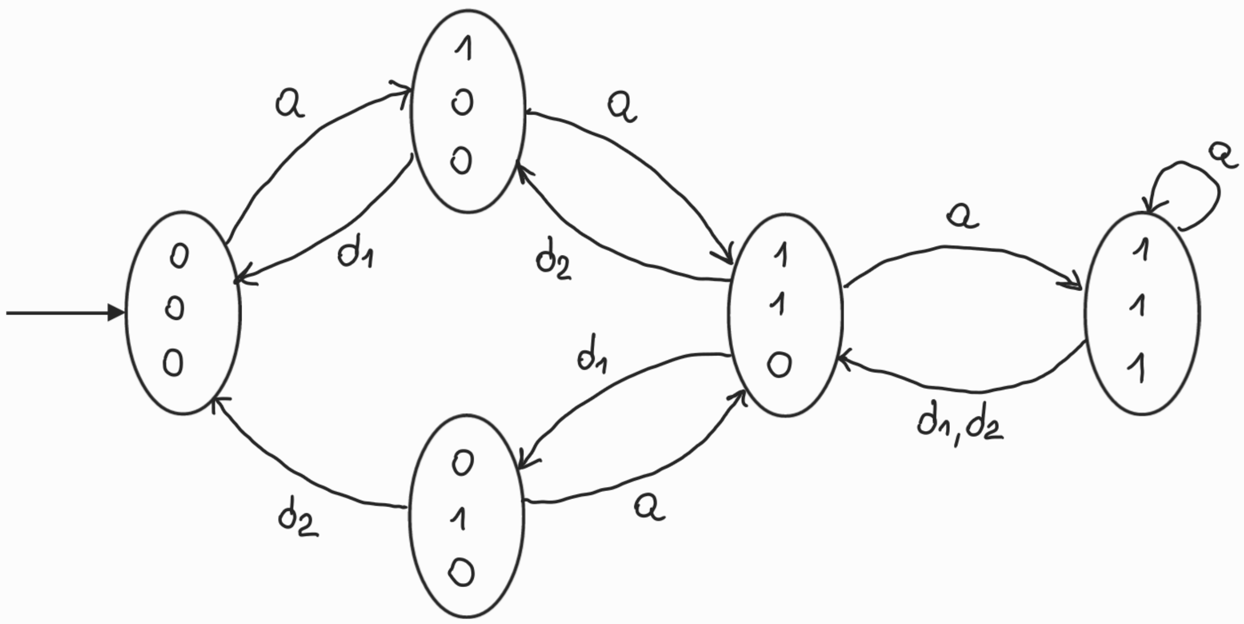
events $\mathcal{E} = \{a, d_1, d_2\}$

- a → arrival of a customer
- d_1 → termination of a service in S_1
- d_2 → termination of a service in S_2

state $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

- x_1 → server S_1: 0 (idle), 1 (working)
- x_2 → server S_2: 0 (idle), 1 (working)
- x_3 → queue: 0 (empty), 1 (full)

Γ, f and x_0 are defined by the following graph (we assume that the system is initially empty):



REMARK: If the two servers are identical, the model can be simplified

⇓

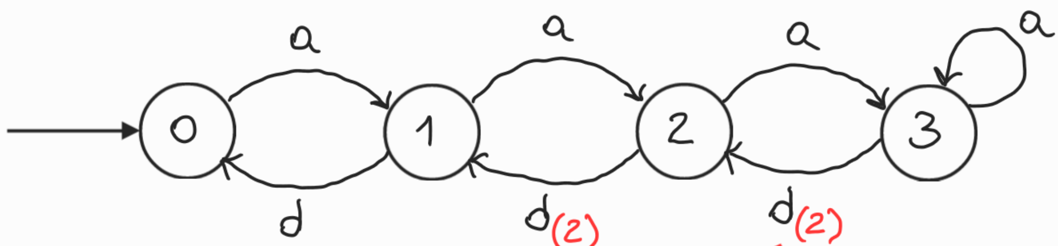
We don't need to distinguish which of the two servers is working

events $\mathcal{E} = \{a, d\}$

- a: arrival of a customer
- d: termination of a service (generic)

state x = # customers in the system $\in \{0, 1, 2, 3\}$

graph:



We use this subscript to remind us that event d may be originated by one of two sources ⇒ useful in the future...