Test of Discrete Event Systems - 30.01.2020

Exercise 1

A mobile robot moves in the rooms of a building. The rooms have doors, that can be of two types: doors connecting two adjacent rooms on the same floor, and lift doors. Connecting doors are always open, and when the robot arrives in front of a connecting door, it instantly enters the other room. When the robot arrives in front of a lift door, it enters the lift if the door is open. Otherwise, it waits for the arrival of the lift, and then enters the lift. When the lift reaches the destination floor, the robot exits the lift and enters the room ahead. It is assumed that opening and closing the lift door, as well as entering and exiting the lift, are instantaneous actions. Moreover, it is assumed that the path in a room starts from a door and ends in front of another door.

1. Assume that the robot moves with constant velocity $v = 0.5$ m/s, and visits 5 rooms, where it covers 20, 12, 28, 15 and 23 m, respectively. The first and the fourth door that the robot encounters, are connecting doors, whereas the second and the third are lift doors. The first lift door is open, while the second is closed, and the lift arrives after 34 s. In both cases, moving to the destination floor takes 25 s. Compute the total time to complete the task.

In the following, assume that the robot moves with constant velocity $v = 0.5$ m/s, and the distance covered in each room is a random variable exponentially distributed with expected value 20 m. When the robot arrives in front of a door, this is a lift door with probability $p = 2/5$, and in this case it is open with probability $q = 1/4$. Moving to the destination floor with the lift always takes 25 s. The waiting time of the lift is uniformly distributed between 15 and 40 s. However, the robot is programmed to wait a maximum amount of time. If the lift has not arrived yet after 30 s, the robot keep moving in the same room. Assume that the robot initially moves in a room.

2. Model the system by a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$.

3. Compute the probability that the second path covered by the robot is on a different floor.

4. Compute the probability that the robot completes paths in at least two different rooms over a time horizon of 45 s.

Exercise 2

A machine produces on demand. If the machine is idle and an order arrives, production starts immediately, provided that the raw parts are already available. Otherwise, a request for raw parts is sent and production starts when the raw parts arrive. Orders arriving when another order is in progress, are ignored. The final product can be either accepted or refused by the customer. In the former case the machine waits for the next order to start a new production. In the latter case, the product is stored, and offered to the next customers, until it is accepted. The machine does not start a new production, as far as there is a stored product. Assume that arrivals of orders are generated by a Poisson process with average interarrival time equal to two hours. Waiting time of raw parts is exponentially distributed with expected value 45 min. Processing time of a final product is exponentially distributed with expected value 75 min. The probability that the raw parts are not available when an order arrives is $p = 3/8$, while the probability that a final product is refused is $q = 1/5$. The machine is initially idle and there is no stored product.

1. Model the system by a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$.
2. Assume that the machine is idle and a product is stored. Compute the average time to selling the product.
3. Assume that an order arrives when the machine is idle and there is no stored product. Compute the probability to return in the same situation before the arrival of another order.
4. Assume that an order arrives when the machine is idle and there is no stored product. Compute the probability to return in the same situation before 90 min, with no order arriving while the machine is busy, and no stored product refused.

Exercise 3

Consider the model of the system of **Exercise 2**. For the following questions, provide numerical answers with the help of Matlab.

1. Compute the probability that the machine is idle one hour after start.
2. Verify the condition $\lambda_{eff} = \mu_{eff}$ for the system at steady-state.
3. Compute the average system time of a customer at steady state.
4. Compute the utilization of the machine at steady state.
5. Compute the probability that a customer has not to wait for the product at steady state.

Exercise 4

A manufacturing company has a setup consisting of two distinct machines, say M_1 and M_2. Each machine produces one component per hour. After production, each component is tested instantly to be identified as defective or nondefective. Let $p = 9/10$ be the probability that a component produced by M_1 is nondefective, and $q = 5/6$ the same probability for M_2. The defective components are discarded, while the nondefective components produced by each machine are stored in two separate bins. When a component is present in each bin, the two are instantly assembled together and shipped out. Each bin can hold at most two components. When a bin is full, the corresponding machine is turned off. It is turned on again when the bin has space for at least one component. Assume that both bins are initially empty.

1. Model the system by a discrete-time homogeneous Markov chain.
2. Compute the probability that neither of the two machines is turned off in the first day.
3. Compute the average time to the first turning off of a machine.
4. Compute the probability that M_1 is turned off at least twice in the first week.