Test of Discrete Event Systems - 02.03.2018

Exercise 1

The system which monitors the pressure of a chemical reactor, can be in one of three states: NORMAL, WARNING, and EMERGENCY. Switching between states is determined by two thresholds $\nu_1 < \nu_2$. The state NORMAL corresponds to the pressure being less than ν_1, while the state EMERGENCY corresponds to the pressure being greater than ν_2. The state WARNING corresponds to the pressure being between ν_1 and ν_2. When the system enters the state EMERGENCY, a valve is opened in order to take instantaneously the pressure back to normal values.

1. Consider the following time profile $p(t)$ for the pressure of the reactor (in bar):

$$p(t) = \begin{cases} 0.25\,t + 0.5 & \text{if } t < 10 \text{ s} \\ 2e^{-(t-10)} + 1 & \text{otherwise.} \end{cases}$$

   Assuming $\nu_1 = 2.0$ bar and $\nu_2 = 3.5$ bar, determine the fraction of time spent in each state over the interval $[0, 20]$ s.

Now assume that the state holding times of the states NORMAL and WARNING follow uniform distributions over the intervals $[5, 30]$ s and $[2, 15]$ s, respectively. In state WARNING, the probability to reach the state EMERGENCY is $q = 1/10$. The time to open the valve is deterministic, and equal to 0.5 s. The system is initially in the state NORMAL.

2. Model the system through a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$.

3. Compute the probability that the system reaches the state EMERGENCY from the state NORMAL, and it takes more than 35 s.

4. Compute the expected value of the recurrence time of the state NORMAL.

Exercise 2

An assembling machine makes finished products from raw parts that are always available. The finished products can be of type 1 with probability $p = 3/5$, and of type 2 otherwise. The assembling time does not depend on the product type, and follows an exponential distribution with expected value equal to 15 min. The machine is followed by a buffer which may host up to two products. When the buffer is full, assembling is suspended. A cart returns to pick up products from the buffer according to a Poisson process with average interarrival time equal to 20 min. When the cart arrives, it picks up all the available products of the demanded type only. The probability that type 1 products are demanded, is $q = 2/3$. The buffer is initially empty.

1. Model the system through a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$.

2. Assume that the buffer is full with one product of both types. Compute the probability that the buffer is empty when the next assembling is completed.

3. Compute the average time that only one type 1 product is available in the buffer.

4. Assume that at time $t = 0$ the buffer is full with one product of both types. Compute the probability that both products are picked up, and no assembling is completed, before time $t = 10$ min.

In questions 5, 6 and 7, it is required to provide **numerical** answers with the help of Matlab.

5. Verify the condition $\lambda_{eff} = \mu_{eff}$ for the buffer at steady-state.
6. Compute the average waiting time of a product in the buffer at steady state.
7. Compute the utilization of the assembling machine at steady state.
8. Discuss at least two solutions to increase the utilization of the assembling machine.

Exercise 3

A working station is formed by three identical resources. In each clock interval, at most one request for use of a resource arrives, with probability $p = 2/5$. If all the resources are busy, the request is rejected. Moreover, in each clock interval, a busy resource terminates the ongoing job with probability $q = 1/3$. All the resources are initially idle.

1. Model the working station through a discrete-time homogeneous Markov chain.
2. Compute the probability that all the resources are busy for at least five consecutive clock intervals.
3. Compute the average number of busy resources at steady state.
4. Compute the average time to have all the three resources busy.
5. Compute the probability that the three resources are never simultaneously busy over the first ten clock intervals.