Test of Discrete Event Systems - 08.02.2018

Please mark with $\times$ the type of test you are willing to take:

i) Endterm (A&QS): Exercises 1 and 2 *[1 hour and 40 minutes]*

ii) Endterm (DES): Exercises 1, 2 and 4 *[2 hours and 20 minutes]*

iii) Full (A&QS): Exercises 1, 2 and 3 *[2 hours and 20 minutes]*

iv) Full (DES): Exercises 1, 2, 3 and 4 *[3 hours]*

Exercise 1

Car accidents occurring on a two-lane highway may obstruct traffic on one lane only or both lanes. Traffic can be restored on one lane at a time. No accidents are observed when only one lane is open, and when both lanes are closed. Assume that both lanes are initially open.

1. When both lanes are open, accidents occur after 24, 18, 30, 36, 14 and 20 hours. The first, third and fourth accident obstruct both lanes. Times to restore one lane are 2, 5, 3, 2, 1.5, 3, 4.5, 4 and 1.5 hours. Compute the mean time between consecutive accidents over the observation period.

Now assume that, when both lanes are open, accidents occur after times following a uniform distribution over the interval $[18, 36]$ hours. An accident obstructs two lanes with probability $p = 2/5$. The time to restore traffic on one lane is deterministic, and equal to 4 hours.

2. Model the system through a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$.
3. Compute the probability that the time between consecutive accidents is greater than 30 hours.
4. Compute the expected value of the time between consecutive accidents.

Exercise 2

A dual-core CPU is formed by two identical parallel processors. If a task arrives and finds one of the processors available, it is executed. Otherwise, it is marked as *rejected*. Execution time of a task follows an exponential distribution with expected value equal to 10 ms. Each rejected task returns independently to the CPU after a time following an exponential distribution with expected value equal to 25 ms. A maximum of two rejected tasks is allowed in the system. For this reason, arrivals of new tasks (i.e., tasks arriving for the first time) are suspended when there are two rejected tasks in the system. Assume that lifetimes of the arrivals of new tasks follow an exponential distribution with expected value equal to 100 ms. At system start, both processors are idle, and no rejected task is present.

1. Model the system through a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0, F)$.
2. Assume that both processors are busy, and one rejected task is present in the system. Compute the probability that both processors terminate the execution of their task before any arrival of other tasks occurs.
3. Compute the average time that both processors are busy and one rejected task is present in the system.

4. Assume that at time $t = 0$ both processors are idle, and one rejected task is present in the system. Compute the probability that no arrivals of new tasks occur, and the rejected task is executed, before time $t = 50$ ms.

Exercise 3

Consider the model of the system of **Exercise 2**. For the following questions, providing **numerical** answers with the help of Matlab will be considered a plus.

1. Compute the probability that both processors are busy 1 s after start.

2. Compute the average number of rejected tasks present in the system 1 s after start.

3. Verify the condition $\lambda_{eff} = \mu_{eff}$ for the CPU at steady-state.

4. Verify the Little's law for the CPU at steady-state.

5. Compute the average utilization of the two processors at steady state.

Exercise 4

Consider a machine which processes parts. Time is discretized into periods of equal length. The machine is idle when it has no part to process. At the end of each period, the machine is supplied with a new part with probability $p = 3/4$. In that case, if the machine can process the part in the next period (because either it is idle, or it has terminated successfully a job in the current period), the new part is accepted. Otherwise, the new part is routed to another machine. Processing of a part takes one period. The first processing of a part may fail with probability $q = 1/10$. In that case, processing is repeated in the next period. Also the second processing may fail with the same probability. In case of a second failure, the machine has to be reset. Reset takes two periods. After reset, the machine is idle.

1. Model the system through a discrete-time homogeneous Markov chain, assuming the machine initially idle.

2. Compute the probability that the machine works continuously and without failures for a number of periods between three and five.

3. Compute the fraction of time that the machine is idle at steady state.

4. Compute the average time between two consecutive resets of the machine.

5. Compute the probability that the machine is never reset over the first 12 periods.