Endterm test of Discrete Event Systems - 20.12.2017

Student: _____

Exercise 1

A robot may perform tasks of four different types, numbered from 1 to 4. Execution of a task can be interrupted by the operator. A new task is started immediately after the termination or the interruption of the previous task. For technical reasons, after a type 1 task, the robot may perform either a type 2 or a type 3 task. After a type 2 task, it may perform either a type 3 or a type 4 task. After a type 3 task, it may perform either a type 2 task or a task of the same type. After a type 4 task, it may perform only a type 1 task.

1. Assume that tasks are performed in the order (1, 2, 3, 3, 2, 4, 1, 3, 2, 4), and the scheduled durations of the tasks are 2.0 and 2.5 min for type 1, 5.0, 3.0 and 5.5 min for type 2, 4.0, 6.0 and 3.5 min for type 3, and 4.5 and 3.0 min for type 4. Moreover, assume that interruptions occur at times 12.0, 23.0 and 25.0 min from start. What tasks are interrupted? What is the total operation time?

Assume that interruptions are disabled, and that, except when the current task is of type 4, the next task is selected randomly, being $q = 2/5$ the probability that the candidate next task with smallest type number is selected. Moreover, assume that durations of type 1 and type 4 tasks are uniformly distributed over the intervals $[2.5, 5.0]$ min and $[3.0, 6.0]$ min, respectively, whereas durations of type 2 and type 3 tasks are deterministic, and all equal to 4.0 min and 3.5 min, respectively.

1. Model the system through a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$, assuming that the first scheduled task is of type 1.
2. Compute the probability that a type 4 task is performed before a type 3 task.
3. Compute the probability that the task sequence (1, 3, 2, 4) is performed, and it takes no more than 15.0 min.

Exercise 2

A warehouse is replenished with goods at time intervals following an exponential distribution with expected value equal to 2 days. Among other goods, three UPS units are available in the warehouse after every supply. Retailers come to stock up on goods at the warehouse according to a Poisson process with average interarrival time equal to 6 hours. The probability that a retailer is willing to buy a UPS unit is $q = 4/15$. The warehouse is initially full.

1. Model the dynamics of the number of UPS units in the warehouse using a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$.
2. Assume that two UPS units are available in the warehouse. Compute the probability that both of them are bought before the warehouse is replenished.
3. Compute the average time that only one UPS unit is available in the warehouse.
4. Compute the probability that there is a daily demand of at least 8 UPS units.