Exam of Discrete Event Systems - 24.09.2014

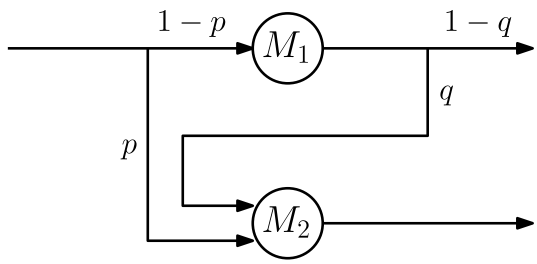
Exercise 1

A channel in the city of Amsterdam has a gate which is normally kept closed and is opened only to allow the passage of boats. Gate opening and closure take 2 minutes each. Both operations cannot be interrupted after they have been started. Boats pass through the gate one at a time. Aim of the gate is to slow down the boats and control the entries in the directions N-S and S-N. Boats in N-S and S-N directions arrive at the gate as generated by Poisson processes with average interarrival times equal to 7.5 minutes and 5 minutes, respectively. When the gate is open, boats pass through it in a time which is uniformly distributed in the interval [30,90] seconds. When a boat has passed through the gate, and there are no boats waiting, the gate is closed; otherwise, the waiting boats are admitted to transit, alternating the two directions if there are waiting boats in both directions. Assume that, at 6 AM (initial time), there are no boats waiting for transit, and consequently the gate is closed.

1. Compute the average number of arrivals of boats during the opening of the gate.
2. Compute the average waiting time for transit of the first boat.
3. Compute the probability that the second boat finds the gate closed.

Exercise 2

Consider the production system depicted in the figure and formed by two machines without queue. When both machines are idle, an arriving part is addressed to M_2 with probability $p = 2/3$, and to M_1 otherwise. If only one machine is idle, the arriving part is addressed to that machine. If both machines are busy, the arriving part is rejected. A part worked in M_1 may need to be worked also in M_2 with probability $q = 1/4$. If M_2 is busy, the part that needs to be worked again in M_2, remains in M_1, and M_1 is not available for a new job. Parts arrive at the production system as generated by a Poisson process with rate 0.5 arrivals/minute. Working times in M_1 and M_2 follow exponential distributions with expected values 1.5 and 1.2 minutes, respectively.



1. Verify the condition $\lambda_{eff} = \mu_{eff}$ for the system at steady state.
2. Compute the utilization of M_1 and M_2 at steady state.
3. Verify the Little's law for M_2.
4. Compute the steady state probability that an arriving part is rejected. Justify the answer.

Exercise 3

A study of the strengths of the football teams of the main American universities shows that, if a university has a strong team one year, it is equally likely to have a strong team or an average team next year; if it has an average team, it is average next year with 50% probability, and if it changes, it is just as likely to become strong as weak; if it is weak, it has 65% probability of remaining so, otherwise it becomes average.

1. What is the probability that a team is strong on the long run?
2. Assume that a team is weak. How many years are needed on average before it becomes strong?
3. Assume that a team is weak. Compute the probability that, along a time horizon of ten years, it is strong at least two years.