# Exam of Discrete Event System - 10.09.2014

## Esercizio 1

A maritime terminal hosts a single container ship at a time. Container trucks arrive at the terminal as generated by a Poisson process with average interarrival time equal to 9 minutes. Trucks enter the ship one at a time, unload the container and leave the terminal. The time required to unload one truck follows an exponential distribution with expected value equal to 6 minutes. Container ships have a maximum capacity of 100 containers. When the maximum load is reached, the ship leaves the terminal and is replaced by another ship which is always available. This replacement requires a time equal to 2 hours. Assume that no truck is initially in the terminal and the ship is empty.

1. Model the terminal through a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0, F)$.
2. Compute the average number of trucks arriving during the replacement phase.
3. Compute the probability that at least 10 trucks arrive during the replacement phase.
4. Compute the average number of trucks arriving during the unloading of a container from the truck.
5. Compute the probability that exactly 4 trucks arrive during the unloading of a container from the truck..
6. Assume that the system is in the initial state. Compute the average waiting time before the first container is loaded onto the ship.

## Esercizio 2

In the café of a big railway station, arriving customers first queue at the cash desk to pay and take the receipt, then queue at the café desk to collect the order. The arrival process of the customers can be modeled as a Poisson process with average interarrival time equal to 45 seconds. Cash desks share only one queue. Issuing the receipt by the operator at the cash desk requires a random time which follows an exponential distribution with expected value 10 seconds. Also the queue for collecting the orders is unique. Service times at the café desk are random and follow an exponential distribution with expected value 35 seconds. The café has four operators. Alternatives for their use are the following three:

*a*) one operator at the cash desk and three at the café desk;

*b*) two operators at the cash desks and two at the café desk;

*c*) three operators at the cash desks and one at the café desk.

The management of the café would like to choose the configuration which minimizes the average system time of a generic customer at steady state.

1. Determine the optimal configuration of the operators.

**Esercizio 3**

In the Dark Ages, the three famous universities $A$, $B$ and $C$ admitted only male students. Assume that each graduate from the three universities had at least one son. Moreover, assume that 80% of the sons of graduates from university $A$ went to university $A$, and the rest to university $B$; 40% of the sons of graduates from university $B$ went to university $B$,and the rest split evenly between the other two; 70% of the sons of graduates from university $C$ went to university $C$, 20% to university $A$, and the rest to university $B$.

1. Provide a Markovian model of the process described above.
2. Compute the steady-state distribution (in %) of the students in the three universities.
3. Given an ancestor graduated from university $B$, compute the probability that no descendant up to the 10th generation attended university $C$.
4. Given an ancestor graduated from university $A$, compute the probability that, during the first ten generations, at least one father and his son attended the same university (not necessarily $A$).