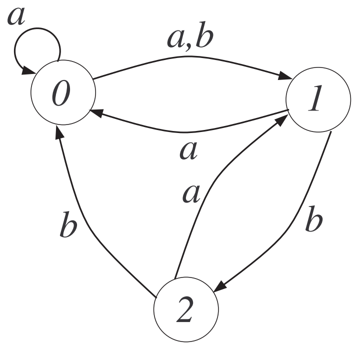
Exam of Discrete Event Systems - 20.02.2014

Exercise 1 (both parts/only first part)

Consider the stochastic timed automaton depicted in the figure.



The lifetimes of events a and b follow uniform distributions in the intervals $[1, 3]$ and $[2, 5]$, respectively. Moreover, $p(0|0, a) = 1/3$.

1. Does the cumulative distribution function of the holding time in state 1 depend or not on the past history before state 1 is reached? Justify the answer through suitable examples.

2. Define Y_0^* the waiting time of the occurrence of the first event. Determine the cumulative distribution function (*optional:* and the expected value) of Y_0^*.

3. Compute the probability to visit all system states in the minimum number of events starting from state 0.

Exercise 2 (both parts/only first part/only second part)

Consider a production system formed by a machine preceded by a unitary buffer. Lifetimes of the arrivals of raw parts in the system follow an exponential distribution with rate $\lambda = 0.2$ arrivals/minute. After every two arrivals, new arrivals are suspended until the occurrence of the next termination of a service in the machine. Lifetimes of the services in the machine follow an exponential distribution as well, with rate $\mu = 0.25$ services/minute.

Both parts/only first part:

1. Model the production system through a stochastic timed automaton.

2. Assume that the system is full and arrivals are possible. Compute the probability that the system is made empty and in the meantime at most one arrival is accepted.

3. Assume that the system is full and arrivals are not possible. Compute the probability that the system is made empty and no arrival is accepted in a time interval $T = 30$ minutes.
Remark: *it is only required to explain the procedure, do not perform computations.*

Both parts/only second part:

4. Verify the condition $\lambda_{eff} = \mu_{eff}$ for the system at steady state.

5. Compare the average system time of a generic part in the system at steady state, with the same value computed for a standard M/M/1/2 queue with arrival and service rates λ and μ defined as above. Provide an explanation of the result of the comparison.

Exercise 3 (both parts/only second part)

In a game of anagrams, at each turn, given a word of three letters formed by using all the letters a, b and c, a new word is generated by choosing two positions $\{i, j\}$, $i, j = 1, 2, 3$, $i < j$, and interchanging the letters in such positions. For instance, if the current word is *abc* and the positions $\{1, 2\}$ are chosen, the next word is *bac*. The probabilities to choose the positions $\{1, 2\}$ and $\{1, 3\}$ are $1/4$ and $1/8$, respectively.

1. Model the game through a discrete-time homogeneous Markov chain, assuming that all words are initially equiprobable.
2. Compute the probability that, starting from the word *abc*, the word *cab* is eventually obtained without in the meantime generating the word *cba*.
3. Compute the probability that, starting from the word *abc*, the same word is eventually again generated not before generating the word *cab*.
4. Compute the average number of words generated to obtain the word *cab* starting from the word *abc*.