Midterm Exam - Discrete Event Systems - 14.11.2013

Exercise 1

Consider a *green garage*, namely a parking where electric vehicles may recharge their batteries. Recharge power is provided by a photovoltaic (PV) plant. All parked vehicles are connected to sockets when they arrive, but only one vehicle at a time receives power. Vehicles are typically recharged in the order of arrival, but in order to ensure a balanced distribution of the power generated by the PV plant among all vehicles, the vehicle on charge is assigned a time slice T. If recharge is not complete when the time slice expires, recharge is interrupted and the vehicle is moved to the end of the queue. If recharge is complete before the time slice expires, the vehicle is removed from the queue and the next vehicle (if present) is assigned with a full time slice.

Power generated by the PV plant during light hours of a winter day is reported in the table below. For the sake of simplicity, it is assumed that power is constant over one hour.

Hour of the day	Power (kW)
8 AM	0.3
9 AM	0.9
10 AM	1.6
11 AM	2.0
12 AM	2.4
1 PM	1.8
2 PM	1.4
3 PM	1.0
4 PM	0.7

The green garage has only three parking places. The first vehicle arrives at 9:30 AM and its battery requires 4.15 kWh to be recharged. The second vehicle arrives at 10:15 AM and its battery requires 3.40 kWh to be recharged. Finally, the third vehicle arrives at 10:40 AM and its battery requires 3.45 kWh to be recharged. All vehicles are picked up by the owners after 5 PM. The time slice is $T = 30$ minutes.

1. Determine whether all vehicles are completely recharged when they are picked up. If the answer is positive, determine also the average total recharge time (total recharge time is the interval between the arrival time of a vehicle and the time when the vehicle is completely recharged). Recall that: *Energy* [kWh] = *Power* [kW] × *Time* [hours].

Then, assume that interarrival times are uniformly distributed in the interval $[0.25, 1.5]$ hours, while net recharge times are uniformly distributed in the interval $[2, 4]$ hours (net recharge time is the time a vehicle actually spends on charge, waiting times excluded). The time slice is still $T = 30$ minutes. Moreover, assume that all vehicles are picked up by the owners as soon as they are completely recharged.

2. Define a logical model $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0)$ of the green garage, assuming that the green garage is empty at 8 AM.

3. Complete the logical model of item 2 with a suitable stochastic clock structure F, and compute $P(E_2 = e)$ and $P(E_3 = e)$ for all events $e \in \mathcal{E}$.

Exercise 2

A machine assembles finite products from raw parts of two types A and B. Assembling includes painting. Products are made by one part of type A and one part of type B, and may differ in the level of paint finish quality. For products of type 1 (which are less expensive) the paint finish quality is poor, while for products of type 2 (which are more expensive) the paint finish quality is very high. The machine is preceded by a buffer with storage capacity of two parts. Assembling of a product does not start until all necessary parts are available. Arriving parts which are not needed to assemble a product, are rejected (for instance, if there is already one part of type A in the buffer, other arriving parts of type A are rejected).

The choice of the product type is random and made at start time of the assembling: with probability $p = 2/3$ the product to assemble is of type 1. Raw parts of type A and B arrive as generated by Poisson processes with rates $\lambda_A = 4$ and $\lambda_B = 5$ arrivals/hour, respectively. Assembling times follow exponential distributions with expected values depending on the product type. The expected assembling time is 20 minutes for type 1 products, and 40 minutes for type 2 products.

1. Model the assembling station through a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$, assuming that the system is initially empty.

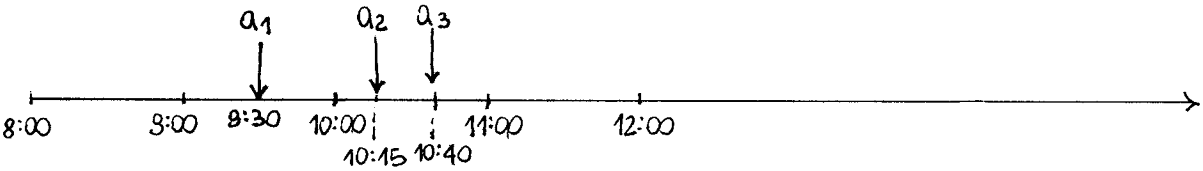
2. Assume that the machine is assembling a type 1 product, and the buffer is full. Compute the probability that the system is empty when a new raw part is accepted in the buffer.

3. Assume that the machine is idle and there is only one type A part (no type B part) in the buffer. Compute the average time to the start of the assembling of the next product.

4. Assume that the machine is assembling a type 1 product, and the buffer is full. Compute the probability that, in a time interval of length $T = 30$ minutes, there are no arrivals of raw parts and two products of type 1 are assembled.

Exercise 1

1. Timing diagram of arrivals:



We denote CAR1 the car which arrives at 9:30, CAR2 the car which arrives at 10:15; and CAR3 the car which arrives at 10:40.
Moreover, we denote R_i the residual energy needed to recharge the battery of CARi, $i=1,2,3$.

TIME INTERVAL	QUEUE	RESIDUAL ENERGY
9:30 - 10:00	(CAR1, -, -)	$R_1 = 4.15 - 0.9 \cdot 0.5 = 3.7$
10:00 - 10:30	(CAR1, CAR2, -)	$R_1 = 3.7 - 1.6 \cdot 0.5 = 2.9$, $R_2 = 3.4$
10:30 - 11:00	(CAR2, CAR1, CAR3)	$R_1 = 2.9$, $R_2 = 3.4 - 1.6 \cdot 0.5 = 2.6$, $R_3 = 3.45$
11:00 - 11:30	(CAR1, CAR3, CAR2)	$R_1 = 2.9 - 2.0 \cdot 0.5 = 1.9$, $R_2 = 2.6$, $R_3 = 3.45$
11:30 - 12:00	(CAR3, CAR2, CAR1)	$R_1 = 1.9$, $R_2 = 2.6$, $R_3 = 3.45 - 2.0 \cdot 0.5 = 2.45$
12:00 - 12:30	(CAR2, CAR1, CAR3)	$R_1 = 1.9$, $R_2 = 2.6 - 2.4 \cdot 0.5 = 1.4$, $R_3 = 2.45$
12:30 - 13:00	(CAR1, CAR3, CAR2)	$R_1 = 1.9 - 2.4 \cdot 0.5 = 0.7$, $R_2 = 1.4$, $R_3 = 2.45$
13:00 - 13:30	(CAR3, CAR2, CAR1)	$R_1 = 0.7$, $R_2 = 1.4$, $R_3 = 2.45 - 1.8 \cdot 0.5 = 1.55$
13:30 - 14:00	(CAR2, CAR1, CAR3)	$R_1 = 0.7$, $R_2 = 1.4 - 1.8 \cdot 0.5 = 0.5$, $R_3 = 1.55$
14:00 - 14:30	(CAR1, CAR3, CAR2)	$R_1 = 0.7 - 1.4 \cdot 0.5 = 0$, $R_2 = 0.5$, $R_3 = 1.55$ at 2:30 PM CAR1 completes recharge
14:30 - 15:00	(CAR3, CAR2, -)	$R_1 = 0$, $R_2 = 0.5$, $R_3 = 1.55 - 1.4 \cdot 0.5 = 0.85$
15:00 - 15:30	(CAR2, CAR3, -)	$R_1 = 0$, $R_2 = 0.5 - 1.0 \cdot 0.5 = 0$, $R_3 = 0.85$ at 3:30 PM CAR2 completes recharge
15:30 - 16:00	(CAR3, -, -)	$R_1 = 0$, $R_2 = 0$, $R_3 = 0.85 - 1.0 \cdot 0.5 = 0.35$
16:00 - 16:30	(CAR3, -, -)	$R_1 = 0$, $R_2 = 0$, $R_3 = 0.35 - 0.7 \cdot 0.5 = 0$ at 4:30 PM CAR3 completes recharge

$\Rightarrow$ At 5PM all vehicles are completely recharged

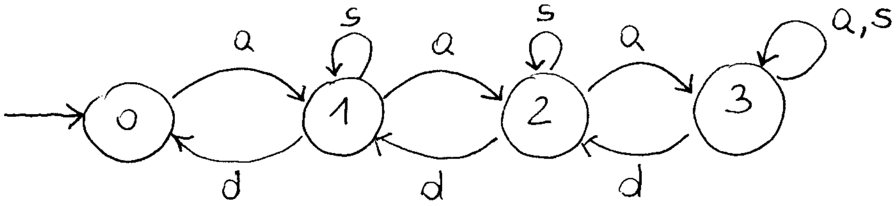
$$\text{AVERAGE TOTAL RECHARGE TIME} = \frac{300 + 315 + 350}{3} \simeq 322 \text{ minutes} = 5 \text{ hours } 22 \text{ minutes}$$

	arrival	recharge	
CAR1:	9:30	14:30	$\Rightarrow$ 300 minutes
CAR2:	10:15	15:30	$\Rightarrow$ 315 minutes
CAR3:	10:40	16:30	$\Rightarrow$ 350 minutes

2. State x = number of vehicles in the green garage $\in \{0,1,2,3\} = \mathcal{X}$.

Events: $\mathcal{E} = \{a, d, s\}$

- a: arrival of a new vehicle
- d: termination of a recharge
- s: expiration of a time slice



3. Lifetimes of event a: $V_a \sim U(0.25, 1.5)$ => uniformly distributed in $[0.25, 1.5]$.

Lifetimes of event d: $V_d \sim U(2,4)$ => uniformly distributed in $[2,4]$

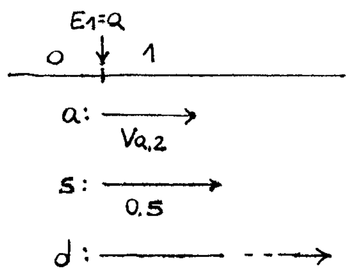
Lifetimes of event s: $V_s = 0.5$ => deterministic

The clock structure F contains the probability distributions of V_a, V_b and V_s.

First of all, observe that the first event is 'a', since the initial state is '0'. Then, the second and third event <u>cannot be</u> 'd', since $V_d \geq 2$, and therefore at least four time slices expire before any vehicle terminates recharge.

=> $P(E_2 = d) = P(E_3 = d) = 0$

$$P(E_2 = a) = P(V_{a,2} \leq 0.5) = \frac{0.5 - 0.25}{1.5 - 0.25} = \frac{1}{5} \qquad \Rightarrow \quad P(E_2 = s) = 1 - \frac{1}{5} = \frac{4}{5}$$



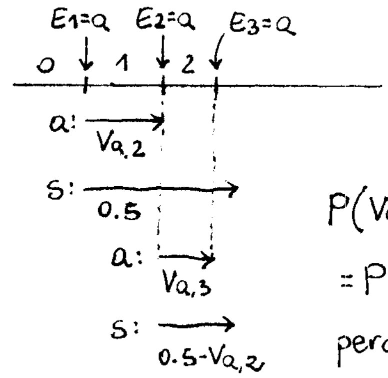
$$P(E_3 = a) = 0 + \frac{2}{5} = \frac{2}{5}$$

2 CASES:

$E_1 = a, E_2 = a, E_3 = a$ (1)

$E_1 = a, E_2 = s, E_3 = a$ (2)

(1)



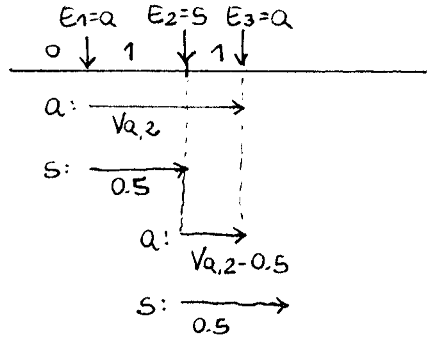
$P(V_{a,2} \leq 0.5, V_{a,3} \leq 0.5 - V_{a,2})$

$= P(V_{a,2} \leq 0.5, V_{a,2} + V_{a,3} \leq 0.5) = 0$

perche' $V_{a,2} \geq 0.25$ e $V_{a,3} \geq 0.25$

=> $V_{a,2} + V_{a,3} \geq 0.5$ sempre

2



$P(V_{a,2} > 0.5,\ V_{a,2} - 0.5 \leq 0.5) =$

$= P(0.5 < V_{a,2} \leq 1.0) = \dfrac{1.0 - 0.5}{1.5 - 0.25} = \dfrac{2}{5}$

$\Rightarrow P(E_3 = s) = \dfrac{3}{5}$

Exercise 2

1\. State $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$x_1 \rightarrow$ machine: 0 (idle), 1 (working product type 1), 2 (working product type 2)

$x_2 \rightarrow$ number of parts of type A in the buffer $\in \{0,1\}$

$x_3 \rightarrow$ number of parts of type B in the buffer $\in \{0,1\}$

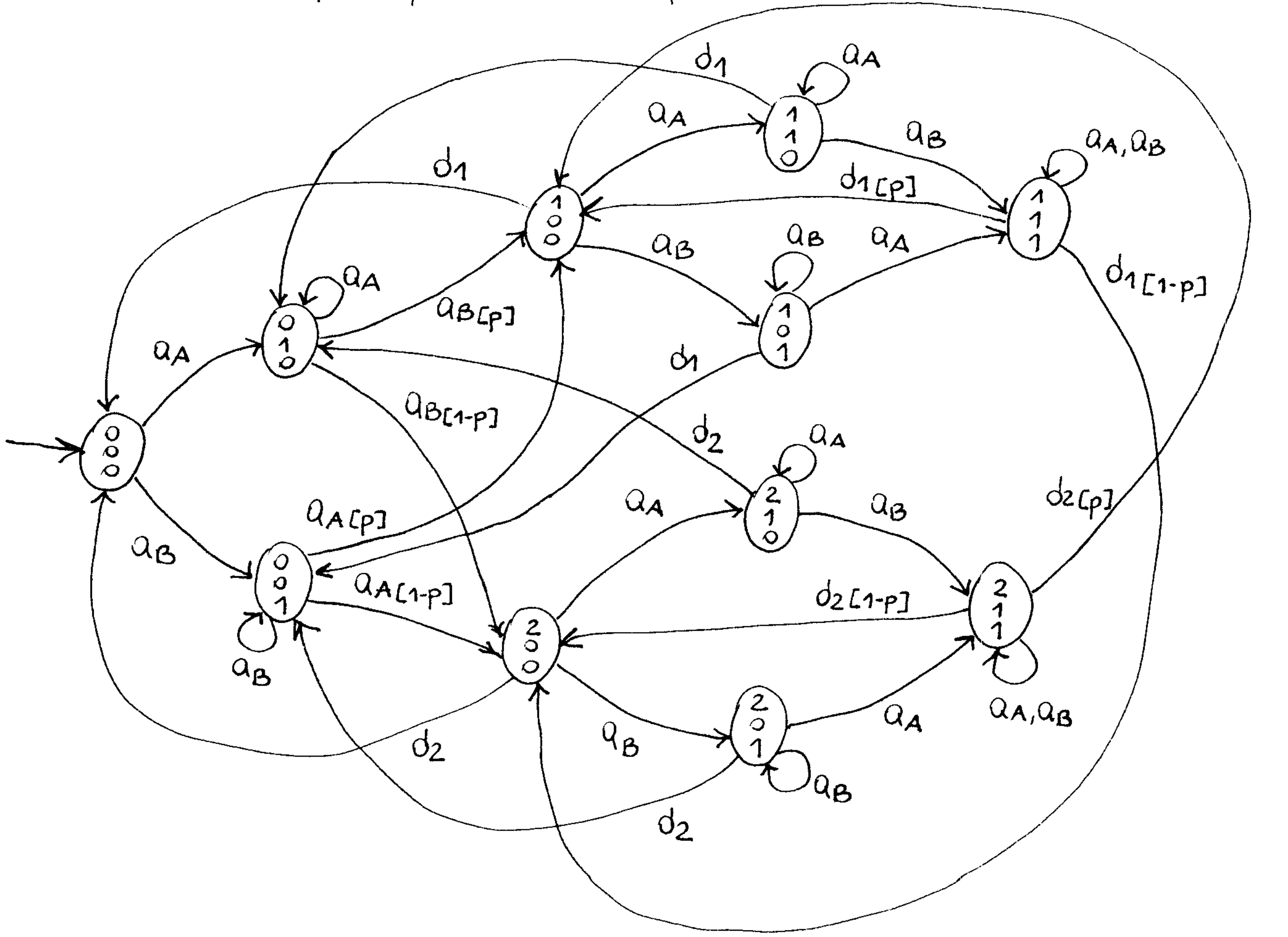
Events $\mathcal{E} = \{a_A, a_B, d_1, d_2\}$

a_A: arrival part type A

a_B: arrival part type B

d_1: termination product type 1

d_2: termination product type 2



$F_{a_A}(t) = 1 - e^{-\lambda_A t}$, $\lambda_A = 4$ arrivals/hour

$F_{a_B}(t) = 1 - e^{-\lambda_B t}$, $\lambda_B = 5$ arrivals/hour

$F_{d_1}(t) = 1 - e^{-\mu_1 t}$, $\mu_1 = 3$ services/hour

$F_{d_2}(t) = 1 - e^{-\mu_2 t}$, $\mu_2 = 1.5$ services/hour

2. current state $X_k = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

$$P(\ldots) = p \cdot \frac{\mu_1}{\lambda_A + \lambda_B + \mu_1} + (1-p) \cdot \frac{\mu_2}{\lambda_A + \lambda_B + \mu_2} = \frac{3}{14} \simeq 0.2143$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xrightarrow{p} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \xrightarrow[d_1]{} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xrightarrow{1-p} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \xrightarrow[d_2]{} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

3. current state $X_k = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. The answer is:

$$\underbrace{E\left[V\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right]}_{\text{holding time in state } \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} = \frac{1}{\lambda_B} = \frac{1}{5} = 0.2 \text{ hours} = 12 \text{ minutes}$$

4. current state $X_k = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. $T = 30$ minutes $= 0.5$ hours.

$$P(\ldots) = P(N_{a_A}(T)=0) P(N_{a_B}(T)=0) \cdot \underbrace{P(V_{d_{1,1}} \leq T, V_{d_{1,1}} + V_{d_{1,2}} \leq T)}_{P(N_{d_1}(T) \geq 2)!} \cdot p$$

$$= e^{-\lambda_A \cdot T} \cdot e^{-\lambda_B \cdot T} \cdot \left[1 - e^{-\mu_1 T} - (\mu_1 T) e^{-\mu_1 T}\right] \cdot p \simeq 0.0033$$