Exam of Discrete Event Systems - 29.07.2013

Student: ____

Exercise 1

A manufacturing station is composed by two series connected machines M_1 and M_2 (M_1 precedes M_2) and a unitary buffer B which serves both M_1 and M_2. Raw parts arrive at the manufacturing station according to a Poisson process with rate $\lambda = 0.1$ arrivals/min. The manufacturing station may host at most two parts simultaneously. Raw parts arriving when the manufacturing station already hosts two parts, are rejected. When M_1 is busy, another part requiring M_1 is temporarily hosted in B. Similarly, when M_2 is busy, another part requiring M_2 is temporarily hosted in B. Each part processed by M_2 may require with probability $p = \frac{1}{4}$ to be processed again starting from M_1. Service times in M_1 and M_2 follow exponential distributions with rates $\mu_1 = 0.2$ and $\mu_2 = 0.25$ services/min, respectively.

1. Model the system through a stochastic state automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, x_0, F)$, assuming that the manufacturing station is initially empty.
2. Given that M_1 and M_2 are both operating, compute the probability that buffer B is full after the next event.
3. Compute the average sojourn time in the state where M_1 and M_2 are both operating.

Exercise 2

Consider the same system as in Exercise 1.

1. Compute the utilization of M_1 at steady state.
2. Compute the average number of parts in the system at steady state.
3. Verify the condition $\lambda_{eff} = \mu_{eff}$ for the system at steady state.
4. Compute the average sojourn time of a part in the system at steady state.

Exercise 3

The wheel of a fair roulette is divided in 37 sectors numbered from 0 to 36. A player starts the game with a capital of \$ 1000. At each round of the game, the player gambles all his capital. If the roulette gives 0, the player receives his bet back. If the roulette gives an even number, the player receives the double of his bet. Otherwise, he loses everything. The player considers himself satisfied and stops to bet in case he reaches a capital of \$ 16000.

1. Compute the probability that the player leaves the game broken (i.e. without money).
2. Compute the average number of bets made by the player.
3. Given that the ball does not land on any odd number in the first four rounds, compute the expected value of the player's capital after the fourth round.