Student: _____

Exercise 1

A machine can be in one of three states: working, under maintenance, out of order. When the machine is working, regular maintenance is scheduled. However, it may happen that the machine fails before the scheduled maintenance. In this case, the machine is labeled as out of order, and a certain time is required to identify the failure. After identification of the failure, the machine is moved to the maintenance state.

Assume that regular maintenance is scheduled after 7 days, and that the time-to-failure of the machine follows a uniform distribution between 5 and 15 days. Moreover, assume that identifying a failure requires a random time uniformly distributed between 1 and 3 days, and that maintenance requires a constant time equal to one day.

- 1. Compute the probability distribution of the sojourn time in the working state.
- 2. Compute the average sojourn time in the working state.
- 3. Compute the average recurrence time of the working state.

Now assume that lifetimes of all the events follow exponential distributions with the same expected values as above.

4. Answer again questions 1, 2 and 3.

Exercise 2

A hardware vendor manufactures \$300 million worth of PCs per year. On average, the company has \$45 million in accounts receivable.

- 1. Compute the average rate of the payments, expressed in dollars per day.
- 2. Compute how much time (expressed in days) elapses on average between invoicing and payment.

Exercise 3

A virus can exist in N different strains, numbered from 1 to N. At each generation the virus mutates with probability $\alpha \in (0, 1)$ to another strain which is chosen at random with equal probability.

1. Compute the average number of generations to find the virus in the same strain.

Let N = 4 and $\alpha = 2/3$.

- 2. Compute the probability that the strain in the sixth generation of the virus is the same as that in the first.
- 3. Assuming that the virus is initially either in strain 1 or 4, compute the probability that the virus never exists in strains 2 and 3 during the first six generations.