Exam of Discrete Event Systems - 29.04.2013

Student: ____

Exercise 1

A canteen is open from 12 AM to 2 PM. From 2 PM customers are no more accepted in the queue, but meal distribution continues for all customers who have entered the queue before 2 PM. Customers are accepted in the queue starting from 11:30 AM. To a first approximation, queue capacity is assumed to be infinite. Customers arrive according to a Poisson process with rate 80 arrivals/hour. Meal distribution is performed by a single operator, whose service times follow an exponential distribution with rate 100 services/hour.

1. Compute the average number of customers who receive lunch every day.

2. Compute the probability that, after the opening of the canteen, exactly five arrivals of customers occur before the distribution of the first meal.

3. Assume that at 2 PM the queue is in practice at steady state. Compute the average time needed to serve all the customers who are still in the queue at 2 PM.

Exercise 2

A small company produces finished products from raw parts through two sequential working phases: phase 2 follows phase 1. Operations in the two phases have random durations following exponential distributions with expected values 1 hour for phase 1, and 46 minutes for phase 2. Raw parts arrive at phase 1 according to a Poisson process with average interarrival time equal to 105 minutes. To a first approximation, it is assumed that both phases have buffers of infinite length. The company has four employees, who can be assigned to working phases as follows:

- one employee to phase 1 and three to phase 2;

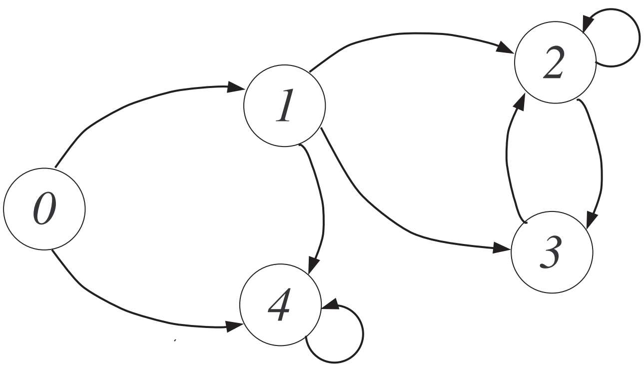
- two employees to phase 1 and two to phase 2;

- three employees to phase 1 and one to phase 2.

1. Determine the assignment of employees to working phases which minimizes the average sojourn time of a part in the system at steady state.

Exercise 3

Consider the discrete-time homogeneous Markov chain whose transition graph is represented in the figure, and with transition probabilities $p_{0,1} = 1/3$, $p_{1,2} = 1/8$, $p_{1,3} = 1/4$ and $p_{2,3} = 4/5$.



1. Compute the average recurrence time for each recurrent state.
2. Compute (without the help of Matlab) the stationary probabilities of all states, assuming the probability vector of the initial state $\pi(0) = [\ 1\ 0\ 0\ 0\ 0\ ]$.