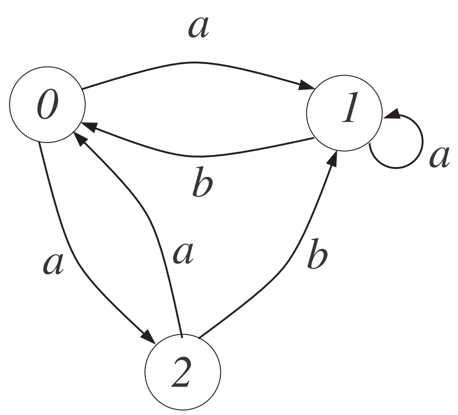
Exam of Discrete Event Systems - 28.02.2013

Student: ____

Exercise 1

Consider the stochastic state automaton in the figure, where a and b are events whose lifetimes follow generic probability distributions, and the initial state is $x_0 = 0$.

1. Given $P(X_2 = 1) = 5/8$ and $P(E_2 = a) = 1/3$, compute the probability $p(1|0, a)$.



Exercise 2

A die is tossed repeatedly. The die is unfair: the probability to obtain any even number is 1/4, whereas the probability to obtain any odd number is 1/12. Suppose to sum the outcomes of the tosses.

1. Assuming to know that the outcome of the first toss is 3, compute the probability that the sum of the first five outcomes is divisible by 4.

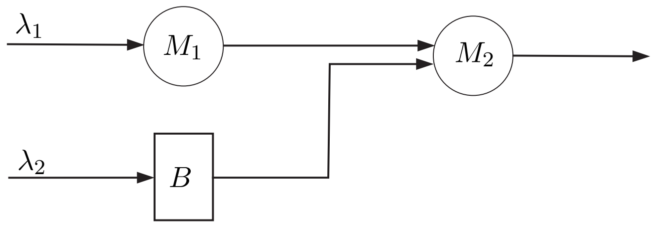
2. Compute the steady-state probability that the sum of the outcomes is divisible by 4.

3. Compute the probability that the sum of the outcomes is never divisible by 4 through the first five tosses.

Suggestion. Use a Markovian model with four states.

Exercise 3

Consider the production system in the figure, composed by two machines M_1 and M_2, and a unitary buffer B. Machine M_2 produces finished products by assembling parts of two types. Parts of type 1 and type 2 arrive as generated by Poisson processes with rates $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$ parts/minute, respectively. Before the assembly, the parts of type 1 are pre-processed in M_1. This task has a random duration which follows an exponential distribution with expected value 4 minutes. Assembly in M_2 has also exponentially distributed random durations with expected value 2.5 minutes. Machine M_2 starts the assembly when parts of both types are available, i.e. when M_1 has terminated pre-processing of a part of type 1 and a part of type 2 is available in B. When M_1 terminates pre-processing of a part of type 1, M_1 holds the part (blocking state) if either M_2 is busy or M_2 is idle but no part of type 2 is available in B. Parts of type 1 arriving when M_1 is busy, are rejected. The same occurs to parts of type 2 arriving when B is full.



1. Provide an appropriate model of the system.
2. Verify the condition $\lambda_{eff} = \mu_{eff}$ for the system at steady-state.
3. Compute the average time which M_1 spends in the blocking state at steady state.
4. Compute the steady state probability that an arriving part is rejected.
5. Assume that M_1 is free, B is full, and M_2 is working. Compute the probability that M_2 starts assembling a new product as soon as it terminates the current task.