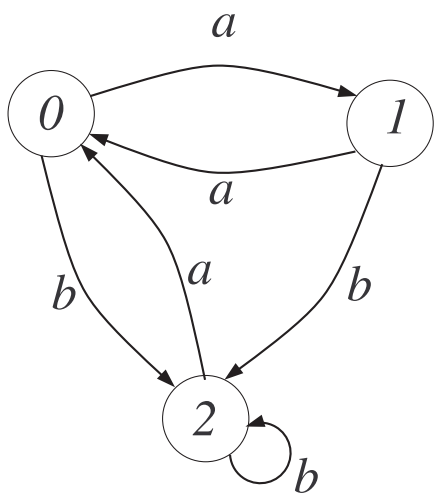
Exam of Discrete Event Systems - 13.02.2013

Student: _____

Exercise 1

Consider the stochastic state automaton represented in the figure, where a is an event with deterministic lifetimes equal to $t_a = 1$, b is an event with lifetimes uniformly distributed in the interval $[0, 3]$, and the initial state is $x_0 = 0$.

1. Compute the probability $P(E_2 = b)$.



Exercise 2

A multi-processor computer is equipped with three identical CPUs. N different processes run on the computer. Each process can be either in the waiting state or in the execution state. The duration of the execution of a process on a CPU is an exponential random variable with expected value 0.77 ms. At the end of the execution, the process is put in the waiting state. A process in waiting state requests again the use of a CPU after a random time which follows an exponential distribution with expected value 1.12 ms. At the moment of the request, if at least one CPU is available, the process is executed. Otherwise, it remains in the waiting state and will request again the use of a CPU as described above.

1. Compute the minimum number N such that the steady-state probability that all the three CPUs are busy, is at least 0.75.

With the value of N computed above:

2. Verify the condition $\lambda_{eff} = \mu_{eff}$ at steady-state for the system composed by the three CPUs.

3. Compute the utilization of a generic CPU at steady state, assuming that, when more than one CPU are available at the moment of a request of execution, the choice is equally probable.

4. Given that two CPUs are busy and one CPU is available, compute the probability that all the three CPUs are available when the next request of execution arrives.

Exercise 3

Define the age of a device as the number of whole days the device has operated. A device of age $j \geq 0$ at the beginning of a day, fails during the day with probability $p_j = jp$, where $p = 0.25$, and in such a case it is replaced by a new identical device, which starts operating at the beginning of the next day.

1. Model the system described above through a discrete time homogenous Markov chain.
2. Compute the probability that a new device operates for the maximum number of days.
3. If the device at day 1 is new, compute the probability that the device at day 10 is new.
4. Compute the average duration (in days) of a device.