Second Partial Exam - Discrete Event Systems - 30.01.2013

Exercise 1

One method of transport used in living cells is axonal transport, in which certain (motor) proteins carry cargo such as mitochondria, other proteins, and other cell parts, on long microtubules. These microtubules can be thought of as the "tracks" of the transportation mechanism. Assume to break the microtubule into $N+1$ equally sized intervals, labelled from 0 (start) to N (end). Every second, the motor protein may have moved to the next interval with probability $p = 1/2$, or back to the previous with probability $q = 1/3$.

1. Model the described transportation system of living cells by a discrete-time homogeneous Markov chain.
2. Determine the maximum N such that the average time from start to end does not exceed 4 seconds.
3. Using $N = 4$, compute the probability that the motor protein never comes back to start.

Exercise 2

Consider a production system composed by a machine preceded by a unitary buffer. The machine may process parts of two types, namely type 1 and type 2. Parts arrive according to Poisson processes with rates $\lambda_1 = 1$ and $\lambda_2 = 0.8$ arrivals/hour, respectively. Arriving parts are rejected if the system is full. When the machine receives a new part, and the new part is of different type than the previous, it must be reconfigured. Duration of a reconfiguration is exponentially distributed with expected value 15 minutes. Processing times of parts of both types are also exponentially distributed with rates $\mu_1 = 1.4$ and $\mu_2 = 1.2$ services/hour, respectively.

1. Model the described production system by a continuous-time homogeneous Markov chain.
2. Assuming that the production system is initially empty, and the machine is configured for parts of type 1, compute the probability that a part arriving after exactly 3 hours is not accepted. Justify the answer.
3. Compute the average reconfiguration time that a generic part has to wait in the machine at steady state.
4. Compute the utilization of the machine at steady state.
5. Verify the condition $\lambda_{eff} = \mu_{eff}$ at steady state.