A: asse di rotazione

C: centro di massa

$$\lambda(x) = \frac{\lambda x}{L}$$

$$M = \int_0^L \frac{\lambda x}{L}\,dx = \frac{\lambda}{L}\frac{L^2}{2} = \frac{\lambda L}{2}.$$

$$x_{CM} = \frac{1}{M}\int_0^L x\,\lambda(x)\,dx = \frac{1}{M}\int_0^L \frac{\lambda}{L}x^2\,dx =$$

$$= \frac{2}{\cancel{\lambda}\cancel{L}}\cdot\frac{\cancel{\lambda}}{\cancel{L}}\cdot\frac{L^{\cancel{3}}}{3} = \frac{2}{3}L.$$

$$I_A = \int d^2\,dm$$

d è pari a x ... quindi:

$$I_A = \int_0^L x^2\frac{\lambda x}{L}\,dx = \frac{\lambda}{L}\int_0^L x^3\,dx =$$

$$= \frac{\lambda}{\cancel{L}}\frac{L^{\cancel{4}\,3}}{4} = \frac{\lambda L^3}{4}.$$

$$I_C = \int d^2\, dm$$

In questo caso $d = |x - \frac{2}{3}L|$ ...

$$I_C = \int_0^L |x - \frac{2}{3}L|^2 \frac{\lambda x}{L} dx =$$

$$= \int_0^L \left(x^2 + \frac{4}{9}L^2 - \frac{4}{3}xL\right)\frac{\lambda x}{L}dx =$$

$$= \frac{\lambda}{L}\int_0^L \left(x^3 + \frac{4}{9}L^2 x - \frac{4}{3}x^2 L\right)dx =$$

$$= \frac{\lambda}{L}\left(\frac{L^4}{4} + \frac{4}{9}L^2\frac{L^2}{2} - \frac{4}{3}L\frac{L^3}{3}\right) =$$

$$= \frac{\lambda}{L}\left(\frac{L^4}{4} + \frac{2}{9}L^4 - \frac{4}{9}L^4\right) = \frac{\lambda}{L}\left(\frac{L^4}{4} - \frac{2}{9}L^4\right)$$

$$= \lambda L^3\left(\frac{1}{4} - \frac{2}{9}\right) = \lambda L^3\left(\frac{9-8}{36}\right) = \frac{\lambda L^3}{36}.$$

Usando il Th. degli assi paralleli:

$$I_A = I_C + \left(\frac{2}{3}L\right)^2 M =$$

$$= \frac{\lambda L^3}{36} + \frac{4}{9}L^2\frac{\lambda L}{2} = \lambda L^3\left(\frac{1}{36} + \frac{2}{9}\right) = \lambda L^3\left(\frac{1+8}{36}\right)$$

$$= \lambda L^3 \frac{9}{36} = \frac{\lambda L^3}{4}.$$

Torna?