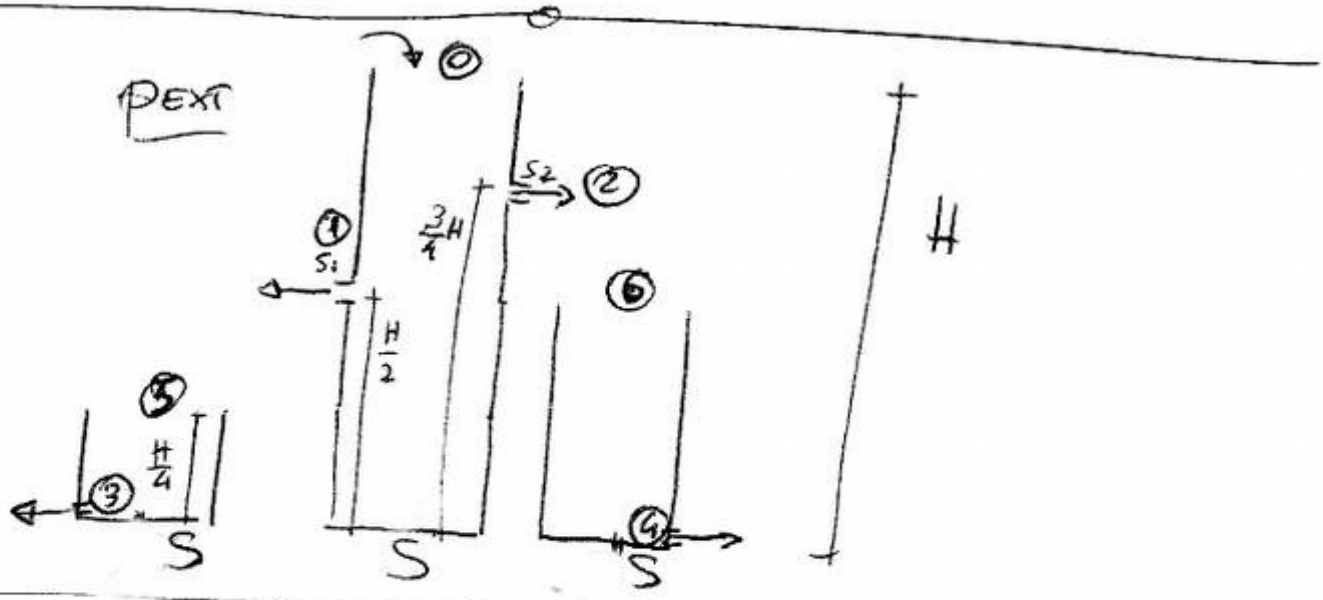
Fisica 1 - 13/02/2017 Esercizio 3



Bernoulli contenitore centrale

(0) $p_{EXT} + \rho g H + \frac{1}{2}\rho v_0^2$

(1) $p_{EXT} + \rho g \frac{H}{2} + \frac{1}{2}\rho v_1^2$

(2) $p_{EXT} + \rho g \frac{3}{4}H + \frac{1}{2}\rho v_2^2$

Equazione di continuità contenitore centrale

$$S v_0 = S_1 v_1 + S_2 v_2$$

Ricavo 3 equazioni

$$\begin{cases} \rho g H + \frac{1}{2}\rho v_0^2 = \rho g \frac{H}{2} + \frac{1}{2}\rho v_1^2 \\ \rho g H + \frac{1}{2}\rho v_0^2 = \rho g \frac{3}{4}H + \frac{1}{2}\rho v_2^2 \\ S v_0 = S_1 v_1 + S_2 v_2 \end{cases}$$

$$\begin{cases} \frac{1}{2}\rho(V_1^2 - V_0^2) = \rho g \frac{H}{2} \\ \frac{1}{2}\rho(V_2^2 - V_0^2) = \rho g \frac{H}{4} \\ S V_0 = S_1 V_1 + S_2 V_2 \end{cases} \Rightarrow \begin{cases} V_1^2 = gH + V_0^2 \\ V_2^2 = \frac{gH}{2} + V_0^2 \\ V_0 = \frac{S_1 V_1 + S_2 V_2}{S} \end{cases}$$

Notare che, poiché $S >> S_1$ e $S >> S_2$, allora

$$\frac{S_1}{S} \simeq \frac{S_2}{S} \simeq 0 \quad \text{e quindi} \quad V_0 \simeq 0$$

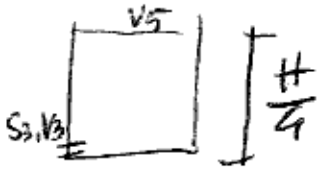
Da cui

$$V_1^2 = gH \qquad V_2^2 = g\frac{H}{2}$$

$$S_1^2 V_1^2 = S_1^2 gH \qquad S_2^2 V_2^2 = \frac{S_2^2 gH}{2}$$

$Q_1 = S_1 V_1 = S_1\sqrt{gH}$ (portata foro 1) $\qquad Q_2 = S_2 V_2 = S_2\sqrt{\frac{gH}{2}}$ (portata foro 2)

Out- sinistra



③ $p_{EXT} + \frac{1}{2}\rho V_3^2$

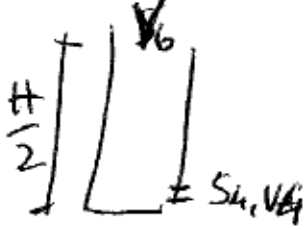
⑤ $p_{EXT} + \rho g \frac{H}{4} + \frac{1}{2}\rho V_5^2$

Continuità $S V_5 = S_3 V_3$

$$\hookrightarrow \begin{cases} \frac{1}{2}\rho V_3^2 = \rho g \frac{H}{4} + \frac{1}{2}\rho V_5^2 \\ V_5 = \frac{S_3 V_3}{S} \simeq 0 \end{cases} \Rightarrow \frac{1}{2}\rho V_3^2 = \rho g \frac{H}{4}$$

$$\boxed{V_3^2 = \frac{gH}{2}}$$

Per quello di destra...



$V_6 = 0$

$$\dots \frac{1}{2}\rho V_4^2 = \rho g \frac{H}{2} \Rightarrow \boxed{V_4^2 = gH}$$

Da cui

$$S_3^2 V_3^2 = S_3^2 \frac{gH}{2} \qquad S_4^2 V_4^2 = S_4^2 gH$$

$Q_3 = S_3 V_3 = S_3 \sqrt{\frac{gH}{2}}$ portata foro 3

$Q_4 = S_4 V_4 = S_4 \sqrt{gH}$ portata foro 4

Notare che:

$Q_1 = Q_3$ e $Q_2 = Q_4$

$$\frac{Q_1}{Q_2} = \frac{S_1\sqrt{g\,H}}{S_2\sqrt{g\,\frac{H}{2}}} = \frac{S_1}{S_2}\sqrt{2}$$

$$\frac{Q_3}{Q_4} = \frac{S_3\sqrt{\frac{g\,H}{2}}}{S_4\sqrt{g\,H}} = \frac{S_3}{S_4}\frac{1}{\sqrt{2}}$$

NOTA $S_3 = S_4$ (dato del problema)

Quindi $\dfrac{Q_3}{Q_4} = \dfrac{1}{\sqrt{2}}$

Ma $\dfrac{Q_1}{Q_2} = \dfrac{Q_3}{Q_4}$. Quindi

$$\frac{S_1}{S_2}\sqrt{2} = \frac{S_3}{S_4}\cdot\frac{1}{\sqrt{2}}$$

$$\boxed{\frac{S_1}{S_2} = \frac{1}{2}} \qquad \boxed{S_2 = 2S_1}$$