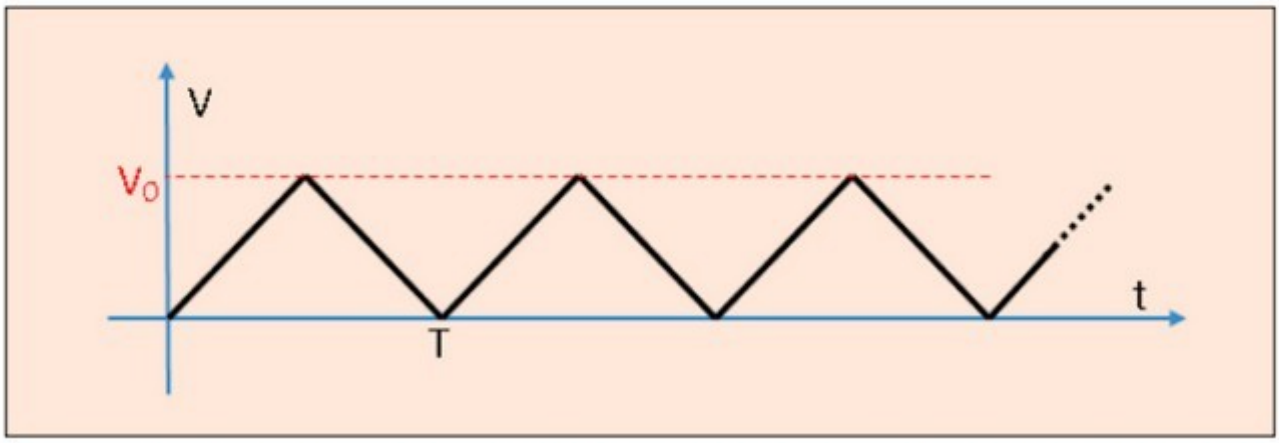
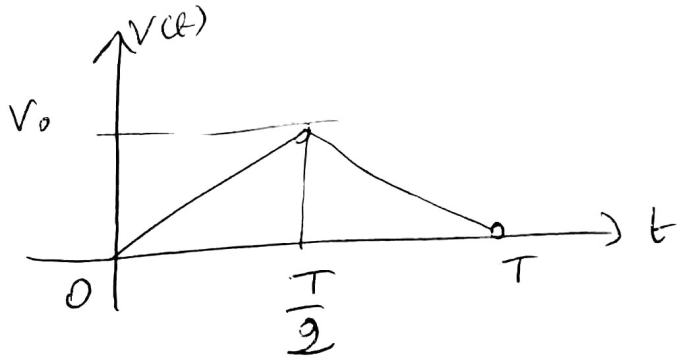
Esercizio estratto dall'esame di Fisica 2 del 4 Novembre 2020



La figura mostra un segnale ad "onda triangolare" oscillante fra $0V$ e $V_0 = 39V$, con periodo $T = 13ms$. Tale segnale viene applicato su un conduttore ohmico la cui resistenza ammonta a 120Ω. Qual è la potenza media dissipata sul conduttore?

La tensione applicata ha periodo T.



$$\int f'(x) f^n(x) dx = \frac{1}{n+1} f^{n+1}(x)$$

$$V(t) = \begin{cases} 2\frac{V_0}{T} \cdot t & 0 < t < \frac{T}{2} \\ 2V_0 - \frac{2V_0}{T} t & \frac{T}{2} < t < T \end{cases}$$

potenza media

$$\overline{P} = \frac{1}{T} \int_0^T \frac{V^2(t)}{R} \cdot dt$$

$$= \frac{1}{RT} \left[\int_0^{\frac{T}{2}} \left(2\frac{V_0}{T} t\right)^2 dt + \int_{\frac{T}{2}}^{T} \left(2V_0 - \frac{2V_0}{T} t\right)^2 dt \right]$$

$$= \frac{1}{RT} \left[\left(\frac{2V_0}{T}\right)^2 \left(\frac{t^3}{3}\right)_0^{\frac{T}{2}} + \frac{T}{2V_0} \int_{\frac{T}{2}}^{T} \left(-\frac{2V_0}{T}\right) \left(2V_0 - \frac{2V_0 t}{T}\right)^2 dt \right]$$

$$= \frac{1}{RT} \left[\left(\frac{2V_0}{T}\right)^2 \frac{T^3}{24} - \frac{T}{2V_0} \frac{\left(2V_0 - \frac{2V_0 t}{T}\right)^3}{3} \Bigg|_{\frac{T}{2}}^{T} \right]$$

$$= \frac{1}{RT} \left\{ \frac{4V_0^2}{T^2} \cdot \frac{T^3}{24} - \frac{T}{6V_0} \left[\left(2V_0 - \frac{2V_0 T}{T}\right)^3 - \left(2V_0 - \frac{2V_0 \frac{T}{2}}{T}\right)^3 \right] \right\}$$

$$= \frac{1}{RT} \left\{ \frac{V_0^2 T}{6} - \frac{T}{6V_0} \left[0 - (V_0)^3 \right] \right\} =$$

$$= \frac{1}{RT}\left\{\frac{V_0^2 T}{6} + \frac{T}{\epsilon V_0} V_0^3\right\} = \frac{1}{RT}\left\{\frac{V_0^2 T}{\epsilon} + \frac{V_0^2 T}{6}\right\}$$

$$= \frac{2\,V_0^2}{R6} = \frac{V_0^2}{3R}.$$