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Optimal Prosumer Storage Management in Renewable Energy Communities Under Demand Response

Gianni Bianchini, Marco Casini * and Milad Gholami

Department of Information Engineering and Mathematics, University of Siena, Via Roma 56, 53100 Siena, Italy; giannibi@diism.unisi.it (G.B.); gholami@diism.unisi.it (M.G.)
* Correspondence: casini@diism.unisi.it

Abstract

This paper deals with the optimal scheduling of prosumers equipped with energy storage facilities within renewable energy communities, and proposes a novel strategy for optimizing storage usage within a price–volume demand response framework. The problem is formulated as a scalable, low-complexity mixed-integer linear program. Furthermore, a heuristic procedure is introduced to ensure redistribution of demand response rewards among participants according to their contribution to achieving demand–response goals. The proposed approach is designed to enhance the benefits for prosumers operating within a community compared to running independently.

Keywords: renewable energy communities; energy storage systems; optimization; demand response

1. Introduction

In order to reduce the environmental impact of energy systems and support the shift to net-zero CO_2 emissions, renewable energy communities (RECs) and demand response (DR) have been recognized as effective solutions.

An REC is formed by a collective of entities (producers, consumers, and prosumers) that participate in energy exchange through the grid [1]. The core objective of an REC is to generate social, economic, and environmental value through optimal renewable energy management, thereby minimizing energy losses and enhancing grid stability. Although national regulations typically define the structure and operation of RECs [2], each autonomous community maintains some flexibility in determining its operational strategies. A key goal of such strategies is to maximize collective welfare by ensuring a fair redistribution of benefits, thus ensuring participation in the REC is always advantageous for all members [3].

On the other hand, DR has proven effective in achieving a balance between generation and demand and in optimizing energy dispatch. Participants in DR programs are economically incentivized to adjust their energy consumption patterns in response to grid operator requests, thus reducing peak demand and improving grid stability [4–7]. In the so-called price–volume DR model, participants receive monetary incentives for adjusting their consumption below certain thresholds during defined time periods. This model of DR has been applied in diverse areas, including, e.g., load forecasting [8,9], smart buildings [10,11], and electric vehicle charging station management [12]. Besides shaping load profiles, optimal management of energy storage systems (ESSs) plays a key role in achieving DR objectives [13,14]. In particular, RECs equipped with ESSs have proven to be a valuable source of DR flexibility [15]. In this context, a common approach is to employ a shared

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BESS, i.e., a single BESS serving multiple prosumers. Such an approach has been proven to enhance efficiency through resource pooling and optimized energy management under specific incentive schemes [16–19]. However, the use of individual BESSs deployed at each prosumer's premises remains a widespread architecture in the context of renewable energy communities, due to its alignment with prosumer autonomy and local incentives [20–23]. Collaborative use of shared energy resources has been explored in several contexts, such as [24,25].

This paper deals with the incorporation of incentive-based DR into the operational strategy of RECs [26], under the assumption that no centralized ESS is present and storage facilities are deployed at the participants' premises. As is commonly done, we assume that the management of physical grid constraints is entrusted to the distribution system operator (DSO) or other external players. The DSO, in order to help satisfy power flow constraints, grid stability, and other services, can define DR programs to be submitted to the REC. In contrast to prior studies, which primarily investigated load profile optimization [27–29], our research demonstrates how DR objectives can be achieved through optimized operation of ESSs. For the special case of all REC entities being pure producers, the problem was addressed in [30], where a strategy for redistributing DR incentives among participants was also proposed. In this paper, we consider the more general case in which the REC is composed of prosumers (i.e., entities that qualify as producers, consumers, or both). This framework calls for a different formulation of the ESS optimization strategy, as well as for a new reward redistribution policy. The proposed method ensures increased economic benefits for prosumers joining the REC compared to standalone operation. The resulting optimization problem turns out to be a low-complexity mixed-integer linear program (MILP) with few binary variables, thus making it computationally feasible to manage large REC memberships. A fairness-oriented heuristics for redistributing the community DR rewards among members tailored to this general case is also developed. Performance evaluation was carried out via numerical simulations on a prototype REC comprising 30 prosumers.

Novelty and Contribution

The main contribution of the paper can be summarized as follows:

- A novel optimization framework is developed for scheduling the operation of distributed storage systems owned by individual prosumers in an REC. The framework accounts for the presence of price–volume signals sent by an external entity (e.g., the distribution system operator) in order to achieve community-level DR objectives. The resulting optimization problem is formulated as a low-complexity MILP.
- As a key theoretical contribution, a rigorous proof is provided that the nonlinear complementarity constraints that arise in order to avoid simultaneous energy import/export and ESS charge/discharge at each individual REC entity are redundant at the optimum. This result allows casting the optimization problem as an MILP with a number of binary variables independent of the REC size, thus making the approach scalable and amenable to the management of large communities.
- The DR reward model proposed in this work extends the one adopted in previous studies such as [30], where a simple saturated ramp function was used to determine the community reward based on the net energy injected into the grid. In contrast, a more realistic trapezoidal reward function that accounts for both under- and over-supply is considered. This extension reflects typical DR program designs, in which over-delivery beyond the specified limits is neither required nor incentivized, and may even be discouraged. As such, the proposed model aligns more closely with practical incentive mechanisms and promotes accurate and effective community-level DR participation.

- A heuristic reward redistribution algorithm is introduced to allocate the DR incentives among REC members. This mechanism ensures that all prosumers receive shares of DR rewards based on their individual contributions to achieving the DR targets and, most importantly, that no prosumer is economically disadvantaged compared to operating independently.

Extensive simulation results are provided to validate the proposed approach, demonstrating its effectiveness in increasing both community-level and individual economic gains.

The paper is organized as follows: the employed REC and DR models are introduced in Section 2. The BESS optimization strategy, as well as a DR reward distribution policy among REC members, are developed in Section 3. Numerical simulations are reported in Section 4, while the conclusions are drawn in Section 5. The symbols used throughout the paper are summarized in Table 1.

Table 1. Nomenclature.

Symbol	Description	Unit
Sets and Indices		
$\mathcal{U}$	Set of prosumers/entities in the REC	–
u	Index for prosumers/entities	–
j	Index for demand response (DR) requests	–
t	Index for time intervals	–
T	Number of time intervals in the optimization horizon	–
T_s	Duration of each time interval	h
$\mathcal{I}(\underline{t}_j, \bar{t}_j)$	Time window of j-th DR request	–
$\mathcal{R}$	DR program, $\mathcal{R} = \{\mathcal{R}_1, \dots, \mathcal{R}_R\}$	–
Optimization Variables		
J_u^*	Total profit of prosumer u under coordinated REC operation with DR	€
Ψ_u^*	Profit of prosumer u in REC without DR reward allocation	€
$J_{u,0}^*$	Optimal profit of prosumer u in standalone operation	€
γ_j	Reward for DR request $\mathcal{R}_j$	€
γ	Total reward for DR program $\mathcal{R}$	€
ξ^*	Total DR reward earned by the REC	€
ξ_u^*	Portion of DR reward allocated to prosumer u	€
$\hat{\xi}_u^*$	Minimum DR reward to ensure $J_u^* \geq J_{u,0}^*$	€
ξ_r^*	Remaining DR reward after satisfying feasibility	€
δ_u	Extra profit for prosumer u from joining REC vs. standalone operation	€
σ_u	Redistribution weight for prosumer u	–
$E_u^c(t)$	Charging energy into BESS of prosumer u at time t	kWh
$E_u^d(t)$	Discharging energy from BESS of prosumer u at time t	kWh
$S_u(t)$	Stored energy in BESS of u at time t	kWh
$E_u^g(t)$	Energy sold to the grid by prosumer u at time t	kWh
$E_u^b(t)$	Energy bought from the grid by prosumer u at time t	kWh
$E_u(t)$	Energy generated by prosumer u at time t	kWh
E_j^{DR}	Total net REC energy reduction during DR request j	kWh
$E^p(t)$	Energy provided by non-scheduled prosumers	kWh
$E^l(t)$	Energy consumed by non-scheduled loads	kWh
$E^n(t)$	Net REC energy injection into the grid	kWh
$z_{r,1}, z_{r,2}, z_{r,3}, z_{r,4}, z_{r,5}$	Binary variables	–

Table 1. *Cont.*

Parameters		
$\overline{E}_u^c$	Max charging rate of BESS of u	kWh
$\overline{E}_u^d$	Max discharging rate of BESS of u	kWh
$\overline{S}_u$	Total BESS capacity of u	kWh
η_u^c	Charging efficiency of BESS of u	–
η_u^d	Discharging efficiency of BESS of u	–
$\overline{E}_u^g$	Max grid selling rate for prosumer u	kWh
$\overline{E}_u^b$	Max grid buying rate for prosumer u	kWh
E_u^{max}	Maximum generation by prosumer u at time t	kWh
E_u^e	Energy demanded by prosumer u at time t	kWh
$\hat{E}_u^{max}$	Forecast of maximum generation by prosumer u at time t	kWh
$\hat{E}_u^e$	Forecast of energy demanded by prosumer u at time t	kWh
$\pi_u^g(t)$	Selling price to the grid for prosumer u at time t	€/kWh
$\pi_u^b(t)$	Buying price from the grid for prosumer u at time t	€/kWh
π_u^s	Unit degradation cost of BESS usage for prosumer u	€/kWh
$\overline{\gamma}_j$	Max DR reward for fully satisfying request j	€
$E_{j,0}^{DR}$	Lower energy threshold for DR reward ramp-up	kWh
$E_{j,1}^{DR}$	Upper energy threshold for DR reward ramp-up	kWh
$E_{j,2}^{DR}$	Lower energy threshold for DR reward ramp-down	kWh
$E_{j,3}^{DR}$	Upper energy threshold for DR reward ramp-down	kWh
ε_u^j	Max discharge energy from BESS of u during DR request j	kWh
$\underline{t}_j$	Start time index of DR request j	–
$\overline{t}_j$	End time index of DR request j	–

2. Problem Formulation and Modeling

The REC model considered in this paper consists of a set of participants, collectively referred to as $\mathcal{U}$. Each participant is a prosumer equipped with a renewable energy generator (e.g., a PV plant) and a BESS, which may or may not include a load. All entities can perform energy exchanges with the main grid. The role of the REC manager is to schedule the operation of these systems through a centralized energy controller. The REC may also include prosumers not equipped with a BESS, as well as entities represented by pure loads. Such entities influence the REC energy balance but are not subject to scheduling.

2.1. REC Model

The REC scheduling system is assumed to run at discrete time instants t with sampling time T_s. Let $\mathcal{T} = \{0, \ldots, T-1\}$ denote the time horizon over which operation is to be optimized, e.g., a whole day. Let $E_u(t) \leq E_u^{max}(t)$ be the (controlled) energy amount generated by entity $u \in \mathcal{U}$ in time slot $t \in \mathcal{T}$, i.e., in the absolute time frame $[tT_s, (t+1)T_s)$, where $E_u^{max}(t)$ denotes the maximum allowed generation at time t, which depends on the installed power and weather conditions. Similarly, denote with $E_u^c(t)$ and $E_u^d(t)$ the command variables representing the energy charged into and discharged from the entity BESS, respectively. The state of each BESS is represented by the stored energy level $S_u(t)$ at the beginning of time slot t. The dynamics of $S(t)$ can be written as

$$S_u(t+1) = S_u(t) + \eta_u^c E_u^c(t) - \frac{1}{\eta_u^d} E_u^d(t), \tag{1}$$

where $0 < \eta_u^c < 1$ and $0 < \eta_u^d < 1$ are the BESS charging and discharging efficiencies, respectively. The command variables $E_u^c(t)$ and $E_u^d(t)$ are bounded by actuation constraints as

$$0 \leq E_u^c(t) \leq \overline{E}_u^c, \quad 0 \leq E_u^d(t) \leq \overline{E}_u^d, \tag{2}$$

where $\overline{E}_u^c$ and $\overline{E}_u^d$ represent the maximum charging and discharging energy per time slot, respectively, while $S_u(t)$ is bounded by the storage capacity $\bar{S}_u$, i.e.,

$$0 \leq S_u(t) \leq \bar{S}_u. \tag{3}$$

For each entity u, let us denote by $E_u^g(t)$ and $E_u^b(t)$ the energy sold to and purchased from the grid in time slot t, respectively. Such quantities are assumed to be bounded as follows:

$$0 \leq E_u^g(t) \leq \overline{E}_u^g, \quad 0 \leq E_u^b(t) \leq \overline{E}_u^b. \tag{4}$$

Moreover, let $E_u^e(t)$ represent the energy demanded by entity u in time slot t. Thus, the energy balance for entity u at any given time slot is then given by

$$E_u^g(t) - E_u^b(t) = E_u(t) - E_u^e(t) - E_u^c(t) + E_u^d(t). \tag{5}$$

The energy amount $E_u^g(t)$ is sold according to a unitary pricing signal $\pi_u^g(t)$, while $E_u^b(t)$ is purchased at a price $\pi_u^b(t)$, both of which are assumed to be known in advance. Moreover, as commonly happens in energy markets, such prices satisfy

$$\pi_u^g(t) \leq \pi_u^b(t). \tag{6}$$

Assuming that entities may either sell or buy energy at any given time (but not both), the following complementarity constraint is in order:

$$E_u^g(t)E_u^b(t) = 0. \tag{7}$$

Additionally, the BESS is assumed to incur a unitary operation cost π_u^s. Then, the net revenue $J_{u,0}$ of entity u over the time horizon $\mathcal{T}$ is expressed as the energy sales revenue, minus the cost for purchased energy and BESS usage, i.e.,

$$J_{u,0} = \sum_{t \in \mathcal{T}} \Big[\pi_u^g(t) E_u^g(t) - \pi_u^b(t) E_u^b(t) - \pi_u^s \big(\eta_u^c E_u^c(t) + \frac{1}{\eta_u^d} E_u^d(t) \big) \Big]. \tag{8}$$

Finally, the total energy provided to the REC by non-scheduled prosumers is denoted by $E^p(t)$, while the overall non-scheduled loads are indicated with $E^l(t)$, so that the net energy injected into the grid by the REC in time slot t reads

$$E^n(t) = \sum_{u \in \mathcal{U}} \Big[E_u^g(t) - E_u^b(t) \Big] + E^p(t) - E^l(t). \tag{9}$$

2.2. Demand Response Model

A price–volume DR model is considered in this work. A DR program $\mathcal{R} = \{\mathcal{R}_j, j = 1, \ldots, R\}$ is composed of a sequence of requests $\mathcal{R}_j$ sent by an external entity, e.g., the distribution system operator, to the REC manager. Each request consists of a time horizon, i.e., a subset of the time frame $\mathcal{T}$, and a monetary reward function. If the net energy injected into the grid by the REC within the specified time horizon lies within specified bounds, then the REC is granted a suitable monetary reward.

Let $\mathcal{I}(\underline{t}_j, \bar{t}_j) = [\underline{t}_j, \bar{t}_j] \subseteq \mathcal{T}$ represent the time horizon related to j-th DR request, and let

$$E_j^{DR} = \sum_{t \in \mathcal{I}(\underline{t}_j, \bar{t}_j)} E^n(t) \tag{10}$$

be the net energy injected into the grid by the REC in such time interval. For the purposes of this work, a DR request $\mathcal{R}_j$ is defined by the tuple

$$\mathcal{R}_j = \left\{ \mathcal{I}(\underline{t}_j, \bar{t}_j),\ \overline{\gamma}_j,\ E_{j,0}^{DR},\ E_{j,1}^{DR},\ E_{j,2}^{DR},\ E_{j,3}^{DR} \right\}, \tag{11}$$

where $\overline{\gamma}_j$ plays the role of the maximum achievable reward and $E_{j,0}^{DR}, E_{j,1}^{DR}, E_{j,2}^{DR}, E_{j,3}^{DR}$ are the energy thresholds that define a piecewise-linear trapezoidal reward function, see Figure 1. As depicted, the function $\gamma_j(E_j^{DR})$ increases linearly between $E_{j,0}^{DR}$ and $E_{j,1}^{DR}$, remains constant at $\overline{\gamma}_j$ between $E_{j,1}^{DR}$ and $E_{j,2}^{DR}$, and decreases linearly to zero between $E_{j,2}^{DR}$ and $E_{j,3}^{DR}$. Therefore, the reward granted to the REC related to $\mathcal{R}_j$ is given by

$$\gamma_j = \begin{cases} 0 & \text{if } E_j^{DR} \leq E_{j,0}^{DR} \\ \dfrac{E_j^{DR} - E_{j,0}^{DR}}{E_{j,1}^{DR} - E_{j,0}^{DR}} \overline{\gamma}_j & \text{if } E_{j,0}^{DR} < E_j^{DR} \leq E_{j,1}^{DR} \\ \overline{\gamma}_j & \text{if } E_{j,1}^{DR} < E_j^{DR} \leq E_{j,2}^{DR} \\ \dfrac{E_{j,3}^{DR} - E_j^{DR}}{E_{j,3}^{DR} - E_{j,2}^{DR}} \overline{\gamma}_j & \text{if } E_{j,2}^{DR} < E_j^{DR} \leq E_{j,3}^{DR} \\ 0 & \text{if } E_j^{DR} > E_{j,3}^{DR} \end{cases} \tag{12}$$

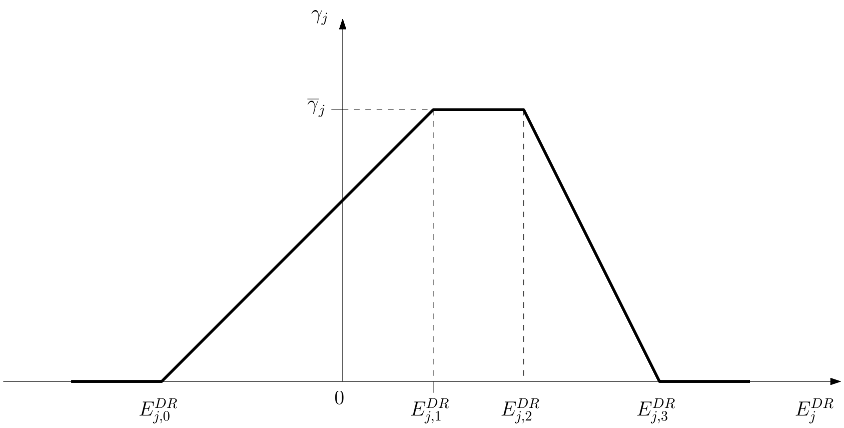

Figure 1. Reward function γ_j pertaining to DR request $\mathcal{R}_j$.

This trapezoidal structure reflects typical incentive-based DR mechanisms, where compensation varies with load reduction to balance grid stability and economic efficiency [31].

Remark 1. *The trapezoidal structure adopted for the DR compensation function γ_j is inspired by real-world DR programs in which compensation increases with the amount of flexibility provided, up to a saturation point. For example, Italy's UVAM (Mixed Enabled Virtual Units) program operated by Terna provides tiered remuneration for flexible capacity, with higher marginal incentives for the first blocks of curtailed energy and a capped payment beyond a defined threshold [32]. Similarly, PJM in the United States implements capacity and energy payments in its DR market, with performance-based adjustments that effectively yield a trapezoidal reward profile [33]. In France, the NEBEF (Notification d'Échanges de Blocs d'Énergie de Flexibilité) mechanism also provides higher marginal* *rates for initial reductions, then flattens payments for larger curtailments [34]. These examples reflect the economic rationale behind trapezoidal schemes, i.e., valuing smaller, reliable-flexibility contributions rather than arbitrarily larger reductions.*

The next result allows reformulating the reward policy (12) associated with each DR request $\mathcal{R}_j$ as a set of linear inequalities involving five binary variables, and follows from the standard big$-M$ argument in optimization.

Proposition 1. *E_j^{DR} and γ_j satisfy (12) if and only if there exist $z_{j,1}, z_{j,2}, z_{j,3}, z_{j,4}, z_{j,5}$ such that*

$$z_{j,1}, z_{j,2}, z_{j,3}, z_{j,4}, z_{j,5} \in \{0,1\}, \tag{13}$$

$$z_{j,1} + z_{j,2} + z_{j,3} + z_{j,4} + z_{j,5} = 1, \tag{14}$$

$$-Mz_{j,1} + E_{j,0}^{DR} z_{j,2} + E_{j,1}^{DR} z_{j,3} + E_{j,2}^{DR} z_{j,4} + E_{j,3}^{DR} z_{j,5} \leq E_j^{DR}, \tag{15}$$

$$E_j^{DR} \leq E_{j,0}^{DR} z_{j,1} + E_{j,1}^{DR} z_{j,2} + E_{j,2}^{DR} z_{j,3} + E_{j,3}^{DR} z_{j,4} + Mz_{j,5}, \tag{16}$$

$$-M(1 - z_{j,1}) \leq \gamma_j \leq M(1 - z_{j,1}), \tag{17}$$

$$-M(1 - z_{j,2}) \leq \gamma_j - \frac{E_j^{DR} - E_{j,0}^{DR}}{E_{j,1}^{DR} - E_{j,0}^{DR}} \overline{\gamma}_j \leq M(1 - z_{j,2}), \tag{18}$$

$$-M(1 - z_{j,3}) \leq \gamma_j - \overline{\gamma}_j \leq M(1 - z_{j,3}), \tag{19}$$

$$-M(1 - z_{j,4}) \leq \gamma_j - \frac{E_{j,3}^{DR} - E_j^{DR}}{E_{j,3}^{DR} - E_{j,2}^{DR}} \overline{\gamma}_j \leq M(1 - z_{j,4}), \tag{20}$$

$$-M(1 - z_{j,5}) \leq \gamma_j \leq M(1 - z_{j,5}), \tag{21}$$

where $M \gg 0$ denotes a constant large enough to avoid inconsistencies in the formulation.

It is assumed that the REC manager retains a fraction $(1 - \alpha)$, $0 < \alpha < 1$, of the total DR reward

$$\gamma = \sum_{j=1}^{R} \gamma_j$$

to cover for the profit of the REC manager itself and for the reward to be granted to other REC participants, i.e., pure loads and non-schedulable prosumers. Therefore,

$$\xi = \alpha\gamma$$

represents the portion of the overall reward γ to be redistributed among the entities in $\mathcal{U}$ according to the policy introduced later on.

3. Optimal REC Management Under DR

In this section, we present the main contribution of this work: the design of an optimal scheduling framework for the operation of BESS resources within an REC under a DR program. This scheduling strategy, to be implemented by the REC manager, aims to maximize a suitable performance index, while satisfying model constraints. Additionally, we propose a fairness-based policy for redistributing DR rewards among participants. The proposed design ensures that participation in the REC is always beneficial for all entities in $\mathcal{U}$, i.e., it guarantees that each participant's total profit from joining the REC is at least as large as the maximum achievable profit stemming from optimally managing their BESS resources in an independent fashion.

3.1. Optimal BESS Management

Given a scheduling time horizon $\mathcal{T}$, the proposed procedure involves three consecutive steps:

1. Calculate the optimal revenue $J^*_{u,0}$ that each entity $u \in \mathcal{U}$ can obtain from energy sales individually, i.e., without participating in the REC DR program.
2. Compute the optimal scheduling of BESS control variables across the REC to maximize the performance index, while ensuring the total community profit is at least

$$J^*_0 = \sum_{u \in \mathcal{U}} J^*_{u,0}. \tag{22}$$

3. Redistribute the DR rewards from Step 2 among participants according to a fairness-oriented heuristics, guaranteeing that each participant's total revenue (energy sales plus DR rewards) is at least $J^*_u \geq J^*_{u,0}$.

Let us define

$$\Psi_u = \sum_{t \in \mathcal{T}} \Big[\pi^g_u(t) E^g_u(t) - \pi^b_u(t) E^b_u(t) - \pi^s_u \big(\eta^c_u E^c_u(t) + \frac{1}{\eta^d_u} E^d_u(t) \big) \Big], \tag{23}$$

which represents the net operation profit of entity u over the time horizon $\mathcal{T}$ arising from energy sales. Moreover, for each entity u, let $\hat{E}^{max}_u(t)$ and $\hat{E}^e_u(t)$, $t \in \mathcal{T}$, denote forecasts of maximum renewable energy generation and demand, respectively, assumed to be available at the beginning of $\mathcal{T}$. The first step of the procedure can be accomplished by solving for each $u \in \mathcal{U}$ the following optimization problem involving the set of decision variables:

$$\Theta_u = \Big\{ \{ E^g_u(t), E^b_u(t), E^c_u(t), E^d_u(t), E_u(t), S_u(t), t \in \mathcal{T} \}, S_u(T) \Big\}. \tag{24}$$

Problem 1.

$$J^*_{u,0} = \max_{\Theta_u} \Psi_u$$

subjected to

$$0 \leq E^c_u(t) \leq \overline{E}^c_u, \quad 0 \leq E^d_u(t) \leq \overline{E}^d_u, \tag{25}$$

$$0 \leq E^g_u(t) \leq \overline{E}^g_u, \quad 0 \leq E^b_u(t) \leq \overline{E}^b_u, \tag{26}$$

$$0 \leq E_u(t) \leq \hat{E}^{max}_u(t), \tag{27}$$

$$S_u(t+1) = S_u(t) + \eta^c_u E^c_u(t) - \frac{1}{\eta^d_u} E^d_u(t), \tag{28}$$

$$0 \leq S_u(t) \leq \overline{S}_u, \qquad \forall t \in \mathcal{T} \tag{29}$$

$$E^g_u(t) - E^b_u(t) = E_u(t) - \hat{E}^e_u(t) - E^c_u(t) + E^d_u(t), \tag{30}$$

$$E^c_u(t) \leq E_u(t), \tag{31}$$

$$S_u(0) = S^0_u, \quad S_u(T) = S^T_u, \tag{32}$$

$$E^c_u(t) E^d_u(t) = 0, \tag{33}$$

$$E^g_u(t) E^b_u(t) = 0. \tag{34}$$

In Problem 1, constraints (25)–(30) are derived from the models in Section 2.1, where the forecast time series $\hat{E}^{max}_u(t)$ and $\hat{E}^e_u(t)$ replace the maximum generation $E^{max}_u(t)$ and demand $E^e_u(t)$, respectively. Constraints (32) set the initial and final storage energy levels to prescribed values (e.g., $S^0_u = S^T_u$ for cyclic operation). Constraints (31) and (33) avoid BESS charging from the grid and simultaneous BESS charging/discharging. Finally, (34) prevents selling and buying energy at the same time. Notice that all constraints in Problem 1 are linear, except (33) and (34). However, the latter turn out to be redundant, as shown by the following result.

Theorem 1. *Constraints* (33) *and* (34) *in Problem 1 are redundant, i.e., they can be relaxed without altering the optimal solution.*

Proof. First, let us focus on (34). It suffices to prove that any optimal solution of the problem obtained by relaxing (34) in Problem 1 is such that $E_u^g(t)E_u^b(t) = 0, \forall t \in \mathcal{T}$. Let Θ_u^* be an optimal solution of the relaxed problem (whose variables are denoted by the superscript *) and assume by contradiction that there exists $\tau \in \mathcal{T}$ such that $E_u^{g*}(\tau) > 0$ and $E_u^{b*}(\tau) > 0$. Let $\widetilde{E}^g = E_u^{g*}(\tau) - \min\{E_u^{g*}(\tau), E_u^{b*}(\tau)\}$ and $\widetilde{E}^b = E_u^{b*}(\tau) - \min\{E_u^{g*}(\tau), E_u^{b*}(\tau)\}$, which yields $\widetilde{E}^g\widetilde{E}^b = 0$. Consider the set of decision variables Θ_u' obtained from Θ_u^* by only substituting $E_u^g(\tau) = \widetilde{E}^g$, $E_u^b(\tau) = \widetilde{E}^b$. It is easy to see that (26) still holds. Moreover, since $\widetilde{E}^g - \widetilde{E}^b = E_u^{g*}(\tau) - E_u^{b*}(\tau)$, constraint (30) is also satisfied, and hence Θ_u' is a feasible solution. By (6), it follows that Θ_u' yields a higher value of the functional (23) than Θ_u^*, which is a contradiction.

A similar argument can be used for constraint (33). Let Θ_u^* be an optimal solution with (33) removed. Suppose there exists $\tau \in \mathcal{T}$, such that $E_u^{c*}(\tau) > 0$ and $E_u^{d*}(\tau) > 0$. Let us define

$$\beta = \min\left\{\eta^d E_u^{c*}(\tau),\ \frac{1}{\eta^c}E_u^{d*}(\tau),\ \frac{E_u^*(\tau)}{\left(\frac{1}{\eta^d} - \eta^c\right)}\right\}.$$

By (31), one has $E_u^*(\tau) \geq E_u^{c*}(\tau) > 0$ and hence $\beta > 0$. Let us define

$$\widetilde{E}^c = E_u^{c*}(\tau) - \frac{1}{\eta^d}\beta\,, \quad \widetilde{E}^d = E_u^{d*}(\tau) - \eta^c\beta\,, \quad \widetilde{E} = E_u^*(\tau) - \left(\frac{1}{\eta^d} - \eta^c\right)\beta.$$

Build a candidate solution Θ_u'' by setting $E_u^c(\tau), E_u^d(\tau), E_u(\tau)$ to $\widetilde{E}^c, \widetilde{E}^d, \widetilde{E}$, respectively, and keeping all other variables as in Θ_u^*. Constraints (25) and (27) follow directly and since $\eta^c\widetilde{E}^c - \frac{1}{\eta^d}\widetilde{E}^d = \eta^c E_u^{c*}(\tau) - \frac{1}{\eta^d}E_u^{d*}(\tau)$, (28) and (29) also hold. Moreover, since $\widetilde{E} - \widetilde{E}^c + \widetilde{E}^d = E_u^*(\tau) - E_u^{c*}(\tau) + E_u^{d*}(\tau)$, constraint (30) is satisfied, and hence Θ_u'' is feasible. Again by (6), Θ_u'' yields a higher objective value than Θ_u^*, a contradiction. □

By Theorem 1, it follows that Problem 1 can be cast as a linear program by removing (33) and (34).

Remark 2. *The computation of $J_{u,0}^*$ in Problem 1 relies on the assumption that each entity $u \in \mathcal{U}$ truthfully discloses its forecast load ($\hat{E}_u^e(t)$), generation ($\hat{E}_u(t)$), and BESS parameters (e.g., $\overline{S}_u, \overline{E}_u^c, \overline{E}_u^d, \eta_u^c, \eta_u^d$) to the REC manager. This assumption is supported by regulatory frameworks, such as the EU Renewable Energy Directive (RED II) [35], which encourage data sharing through contractual agreements, to ensure efficient energy management and compliance with grid requirements. However, this assumption warrants critical examination due to potential incentives for entities to misreport data to maximize their individual profits, e.g., by inflating generation forecasts or BESS capacities to increase their baseline profit $J_{u,0}^*$, which could impair reward distribution. To mitigate the risk of misreporting, the REC manager can implement robust verification mechanisms. For instance, smart meter data can be used to validate load and generation profiles [31], while third-party audits or manufacturer certifications can verify BESS parameters [36]. Technologies such as blockchain can further enhance transparency and security in data sharing by providing an immutable record of reported parameters and energy transactions [37]. Additionally, contractual agreements within the REC can include penalties for misreporting, such as reduced DR reward allocations or exclusion from future DR programs, thereby aligning individual incentives with collective goals.*

In what follows, it is assumed without loss of generality that the REC entities are dimensioned, so that Problem 1 is always feasible for all $u \in \mathcal{U}$. For instance, this is the case if the maximum load does not exceed the maximum power that can be provided by the grid, i.e., $\max_{t\in\mathcal{T}} \hat{E}^e_u(t) \leq \overline{E}^b_u$, $\forall u \in \mathcal{U}$, which is a common design requirement.

Once the optimal solution of Problem 1 has been computed for all entities, the sought optimal storage scheduling strategy is obtained via the solution of the following optimization problem, where the set of decision variables is defined as

$$\Theta = \left\{\Theta_u, \gamma_j, z_{j,1}, z_{j,2}, z_{j,3}, z_{j,4}, z_{j,5}, \forall u \in \mathcal{U},\ \forall j = 1, \dots, R\right\}. \tag{35}$$

Problem 2.

$$J^* = \max_{\Theta} \sum_{u\in\mathcal{U}} \Psi_u + \alpha \sum_{j=1}^{R} \gamma_j$$

subjected to

$$(25) - (32),\ \forall u \in \mathcal{U},\ \forall t \in \mathcal{T} \tag{36}$$

$$(9), (10), (13) - (21),\ \forall j = 1, \dots, R \tag{37}$$

with Ψ_u *as in* (23).

In Problem 2, the objective function to be maximized represents the overall REC profit (net operation profit plus DR rewards), while (36) defines the operating constraints in the same fashion as Problem 1, and (37) enforces the DR rewarding policy. To avoid complicating the notation, constraint (9) in Problem 2 is assumed to be evaluated for $E^p(t)$ and $E^l(t)$ equal to suitable forecasts $\hat{E}^p(t)$ and $\hat{E}^l(t)$ of the respective variables.

The following result establishes the existence of a feasible solution to Problem 2. Moreover, it shows that the optimal total profit J^* obtained by solving Problem 2 is no less than the cumulative profit J_0^* in (22) attained by optimally operating each individual entity in an autonomous fashion via Problem 1.

Theorem 2. *Problem 2 is feasible. Moreover, the optimal solution satisfies* $J^* \geq J_0^*$.

Proof. For each entity $u \in \mathcal{U}$, let Θ^*_u be the optimal solution of Problem 1, with objective $J^*_{u,0}$. Let us consider a candidate solution to Problem 2 of the form

$$\widetilde{\Theta} = \left\{\widetilde{\Theta}_u, \widetilde{\gamma}_j, \widetilde{z}_{j,1}, \widetilde{z}_{j,2}, \widetilde{z}_{j,3}, \widetilde{z}_{j,4}, \widetilde{z}_{j,5}, \forall u \in \mathcal{U},\ \forall j = 1, \dots, R\right\},$$

where $\widetilde{\Theta}_u = \Theta^*_u$, $\forall u \in \mathcal{U}$, while $\widetilde{\gamma}_j$, $\widetilde{z}_{j,1}, \widetilde{z}_{j,2}, \widetilde{z}_{j,3}, \widetilde{z}_{j,4}, \widetilde{z}_{j,5}$, $\forall j = 1, \dots, R$ are to be defined. Since Θ^*_u satisfies (25)–(32), constraints in (36) hold. Let us consider the j-th DR request. Since $\widetilde{\Theta}_u$ is given, the energy injected into the grid E^{DR}_j is obtained through (9)–(10). Defining $E^{DR}_{j,-1} = -\infty$, $E^{DR}_{j,4} = \infty$, from the DR program definition (11), it follows that there exists $k \in \{1, 2, 3, 4, 5\}$ such that $E^{DR}_{j,k-2} < E^{DR}_j \leq E^{DR}_{j,k-1}$. For each $j = 1, \dots, R$, set $\widetilde{z}_{j,k} = 1$ and $\widetilde{z}_{j,w} = 0, w \neq k$, and correspondingly $\widetilde{\gamma}_j$ according to (12) in $\widetilde{\Theta}$. The resulting candidate solution $\widetilde{\Theta}$ yields $\Psi_u = J^*_{u,0}$, and moreover $\widetilde{\gamma}_j \geq 0$, $j = 1, \dots, R$, then by (22) it follows that the objective function $\widetilde{J}$ corresponding to $\widetilde{\Theta}$ is such that $\widetilde{J} \geq J_0^*$, and hence $J^* \geq \widetilde{J} \geq J_0^*$. Moreover, it is easy to check that (13)–(21) are also satisfied, and therefore $\widetilde{\Theta}$ is a feasible solution of Problem 2. □

The optimal solution Θ^* of Problem 2 defines the optimal scheduling of the REC storage resources of all entities $u \in \mathcal{U}$. Note that, for each entity u, the value Ψ^*_u of Ψ_u at the optimum of Problem 2 represents the net operation profit under the optimal scheduling, while the overall optimal DR reward, which is to be shared among all entities in $\mathcal{U}$, is given by

$$\xi^* = \alpha \sum_{j=1}^{R} \gamma_j^*, \tag{38}$$

where γ_j^* denotes γ_j at the optimum of Problem 2.

Remark 3. *In Problem 2, the complementary constraints in (33) and (34) are omitted, since they are redundant. Such a property can be easily proved by following a similar argument as in the proof of Theorem 1. As a consequence, Problem 2 turns out to be a MILP with $5R$ binary variables (see Proposition 1). Notably, the number of binary variables is independent of the cardinality of $\mathcal{U}$, which makes the complexity of Problem 2 scale well with the REC size, as also shown in the numerical simulations in Section 4. More specifically, the computational complexity is polynomial in the cardinality of $\mathcal{U}$ and in general exponential in R. Since one and only one $z_{j,i}$ is equal to 1 for a given j (see (14)), an instance of Problem 2 involves the solution of 5^R LPs in the worst case (though modern MILP solvers exploit the complementarity constraint more efficiently). In practical DR programs, however, R is typically a few units, due to grid operator constraints and operational feasibility.*

3.2. Reward Distribution Policy

Now, let us focus on the policy for sharing the DR reward ξ^* among the prosumers. For this purpose, let ξ_u^* denote an allocation of ξ^* among the entities of $\mathcal{U}$, i.e.,

$$\xi^* = \sum_{u \in \mathcal{U}} \xi_u^*, \quad \xi_u^* \geq 0, \tag{39}$$

so that the total profit of entity $u \in \mathcal{U}$ in the presence of DR amounts to $J_u^* = \Psi_u^* + \xi_u^*$. Such a redistribution is deemed feasible if the total profit of every prosumer amounts to no less than the optimal profit when acting individually, that is

$$J_u^* \geq J_{u,0}^*, \quad \forall u \in \mathcal{U}. \tag{40}$$

To guarantee (40), at least the quantity $\hat{\xi}_u^* = J_{u,0}^* - \Psi_u^*$ needs be assigned to all prosumers. The remaining reward amount

$$\xi_r^* = \xi^* - \sum_{u \in \mathcal{U}} \hat{\xi}_u^* \tag{41}$$

must then be distributed according to a suitable fairness-oriented heuristics. To this purpose, let us define for all $j = 1, \ldots, R$ and $u \in \mathcal{U}$

$$\varepsilon_u^j = \min\left\{ \left(\sum_{t=0}^{\underline{t}_j - 1} \min\{E_u(t), \overline{E}_u^c\} \right) - \sum_{k=0}^{j-1} \varepsilon_u^k,\ (\overline{t}_j - \underline{t}_j)\overline{E}_u^d,\ \overline{S}_u \right\}, \tag{42}$$

where $\varepsilon_u^0 = 0$ is additionally assumed. Then, we propose to partition ξ_r^* according to the weights σ_u defined as

$$\sigma_u = \sum_{j=1}^{R} \left(\varepsilon_u^j \cdot \frac{\overline{\gamma}_j}{\overline{E}_j^{DR} - \underline{E}_j^{DR}} \right), \quad \forall u \in \mathcal{U}. \tag{43}$$

Therefore, the total amount of DR reward assigned to $u \in \mathcal{U}$ is given by

$$\xi_u^* = \hat{\xi}_u^* + \frac{\sigma_u}{\sum_{k \in \mathcal{U}} \sigma_k} \xi_r^*, \tag{44}$$

and the extra profit obtained by entity u from joining the REC with respect to optimal standalone operation amounts to

$$\delta_u = J_u^* - J_{u,0}^* = \frac{\sigma_u}{\sum_{k \in \mathcal{U}} \sigma_k} \xi_r^*.$$

The reward distribution heuristic outlined above is aimed at guaranteeing higher benefits to the entities which contribute the most to achieving the community DR reward by suitably managing their BESS. The rationale behind such a policy is as follows: First, the quantity $\hat{\xi}_u^*$ is assigned to all entities to satisfy (40) at equality. Then, the remaining profit ξ_r^* is shared among prosumers to guarantee an individual extra profit proportional to the potential expenditure of BESS usage on achieving the REC-wide reward over the entire DR program. To see that this is indeed the case with the proposed redistribution policy, consider the first DR request $\mathcal{R}_1$: the quantity ε_u^1 in (42) represents the maximum energy that can be discharged from the BESS into the grid during the DR time window $\mathcal{I}(\underline{t}_1, \overline{t}_1)$, which in turn equals the minimum among the generation up to time $\underline{t}_1$, the maximum energy which can be discharged in the first DR period considering the BESS discharging rate, and the total BESS capacity. For every subsequent DR request $\mathcal{R}_j$, a similar computation is performed, the only difference being the nonzero term $-\sum_{k=0}^{j-1} \varepsilon_u^k$ in (42), which guarantees that ε_u^j only accounts for the energy availability during $\mathcal{I}(\underline{t}_j, \overline{t}_j)$. Then, the redistribution weights σ_u, $u \in \mathcal{U}$, are computed as weighted sums of ε_u^i over all DR requests, as in (43). Since DR requests may differ by both the total reward and the involved energy levels, each ε_u^i in (43) is normalized by the ratio $\overline{\gamma}_j / (\overline{E}_j^{DR} - \underline{E}_j^{DR})$.

Remark 4. *The reward distribution policy ensures that prosumers that sacrifice individual energy sales profit (Ψ_u^*) to meet DR objectives are fully compensated, as the total profit $J_u^* = \Psi_u^* + \xi_u^*$ is guaranteed to be at least as large as the standalone profit $J_{u,0}^*$ (40). The minimum reward $\hat{\xi}_u^* = J_{u,0}^* - \Psi_u^*$ compensates for any shortfall in Ψ_u^* due to BESS adjustments for DR, while the remaining reward ξ_r^* is allocated based on weights σ_u (43), which reflect each prosumer's contribution to DR through BESS discharge capacity (ε_u^j). This ensures that each participant prosumer is rewarded according to their verified contributions to collective DR goals. To address potential bad faith behavior, such as misreporting BESS capacity or generation forecasts to inflate ε_u^j or $J_{u,0}^*$, the REC manager can leverage verification mechanisms outlined in Remark 2, including smart meter data validation [31], third-party audits [36], and blockchain technology [37]. Additionally, contractual penalties within the REC framework, such as reduced reward allocations for misreporting, promote the alignment of individual incentives with collective goals, thus discouraging bad faith actions.*

4. Numerical Simulations

A simulation study, consisting of two examples, was conducted to evaluate the performance and computational feasibility of the proposed method. The first example was an illustrative toy example with three entities, designed to show the rationale behind the reward redistribution procedure. The second example was a realistic example with 30 prosumers, which demonstrated the scalability and effectiveness of the method under real-world conditions. Since we are focusing on BESS management, we explicitly considered only prosumers and producers equipped with an energy storage system, while consumers, prosumers, and producers without BESS were treated in aggregated form through an overall community load and generation profile. The time horizon used in the optimization problems for both examples was set to one day, with a sampling time of 15 min, i.e., $\mathcal{T} = \{0, \ldots, 95\}$. A total of 30 days, corresponding to the month of June, were simulated. We assumed that reliable forecasts of generation and load profiles were available. The generation profiles for the PV prosumers were derived from real data provided by the Photovoltaic Geographical Information System (PVGIS) [38], which offers high-resolution solar irradiance and PV generation data for the European region, tailored to the specified peak energy capacities $\overline{E}_u$ of each entity. The load profiles were based on typical residential and commercial consumption patterns from publicly available datasets provided by the Italian transmission system operator, Terna [39]. For each entity u, the initial and final energy stored in the BESS were set to $S_u^0 = S_u^T = 0$, and the charging and discharging efficiencies were $\eta_u^c = \eta_u^d = 0.95$. The energy selling price $\pi_u^g(t)$ was obtained from the Italian energy market [40] under the *Ritiro Dedicato* scheme defined by GSE [41]. The energy buying price $\pi_u^b(t)$, defined by the *Servizio di Maggior Tutela* from ARERA [42], was also time varying, reflecting dynamic pricing for energy imported from the grid. The operational cost of the storage system was $\pi_s = 0.01\,€/\text{kWh}$. This value represents an average estimate of the ESS operational cost, accounting for battery degradation over its cycle life, based on a review of recent literature [20,43,44].

Two DR requests were considered for each day, one in the morning and one in the afternoon, with random start time and duration. This choice of $R = 2$ reflected a realistic scenario, as practical DR programs, such as those in the Italian energy market [40], typically limit daily DR requests to 1–4 due to grid operator constraints and operational feasibility.

The parameter α, set to 0.9 in this study, represents the portion of DR reward which is redistributed among the prosumers in $\mathcal{U}$, as dictated by the REC's policy. This value ensures that the majority of the DR reward is allocated to prosumers that actively contribute to achieving DR objectives through their BESS, while the remaining fraction $(1 - \alpha) = 0.1$ supports the REC's operational sustainability. The energy bounds and rewards of DR requests were dynamically computed on the basis of the load and generation profiles of each day. Specifically, the lower bound $E_{j,0}^{DR}$ was chosen as the energy injected into the grid assuming no BESS operation. This guaranteed that DR rewards were granted only if entities operated their batteries to reduce consumption with respect to the baseline.

4.1. Toy Example

To provide an intuitive explanation of the reward redistribution algorithm, we considered a simplified case with three PV producers equipped with BESS. The peak PV generation capacity $\overline{E}_u$ and BESS capacity $\overline{S}_u$ of each producer were chosen as $\overline{E}_1 = 22\,\text{kWh}$, $\overline{E}_2 = 18\,\text{kWh}$, $\overline{E}_3 = 10\,\text{kWh}$, $\overline{S}_1 = 24\,\text{kWh}$, $\overline{S}_2 = 16\,\text{kWh}$, and $\overline{S}_3 = 9\,\text{kWh}$.

In Figure 2 (top), we report the PV generation profiles of entities #1, #2, and #3 for June 1. The middle panel shows the energy each entity sold to the grid over the day, while the bottom panel depicts the BESS stored energy. The figure shows that during DR periods, all entities discharged their BESS to help fulfill the DR request, taking advantage of the higher reward opportunities. Figure 3 (top) compares the cumulative daily profit in standalone operation versus REC participation. This illustrative case confirmed that cooperation led to a higher total profit. The bottom panel shows the DR reward at the community level compared to the theoretical maximum. Figure 4 (top) shows the cumulative energy sold within DR periods by each entity, where entity #1 contributed the most within DR intervals, followed by entity #2 and then #3, and the bottom panel reports the corresponding extra profit δ_u, which was proportional to their potential DR contributions as captured by σ_u. This example demonstrates the logic and outcome of the proposed redistribution mechanism, which rewards higher contributors with great potential (e.g., large ESS), while still ensuring no participant is worse off than in the standalone case, thus allowing all participants to benefit from collective coordination.

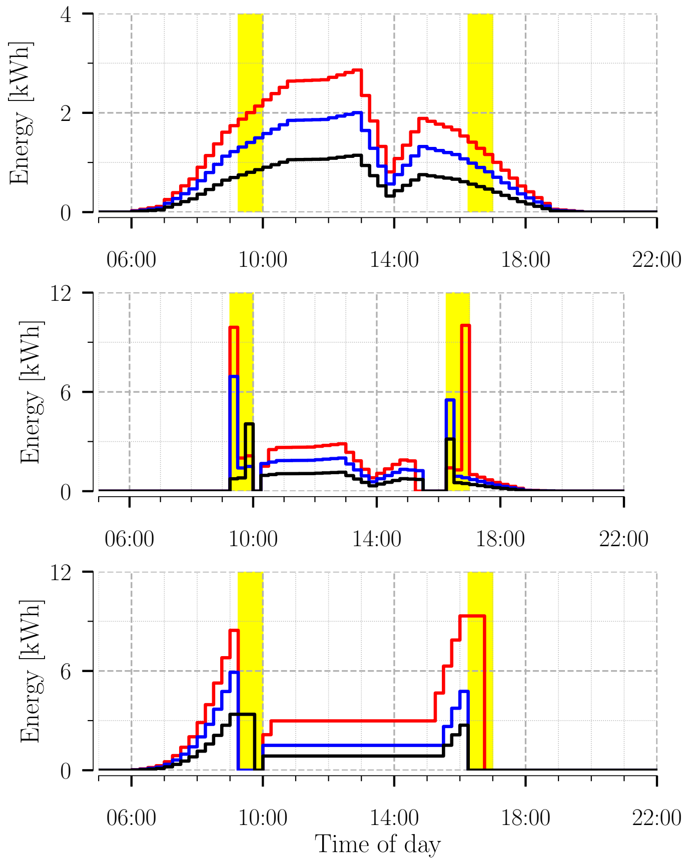

Figure 2. Toy example: [**Top**] generation profile $E_u(t)$ for entity #1 (red), #2 (blue) and #3 (black). [**Middle**] Sold energy $E_u^g(t)$ injected into the grid at REC level by entity #1 (red), #2 (blue) and #3 (black). [**Bottom**] Energy stored in the BESS $S_u(t)$ by entity #1 (red), #2 (blue) and #3 (black). Yellow bars denote the time periods of DR requests.

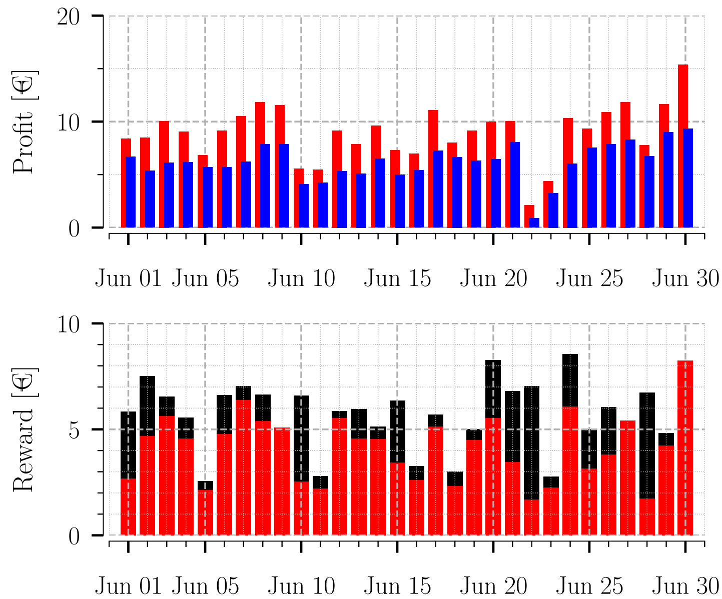

Figure 3. Toy example: [**Top**] cumulative daily profit of all entities operating independently J_0^* (blue) and joining the REC J^* (red). [**Bottom**] Daily DR reward at REC level (red), and maximum achievable reward (black).

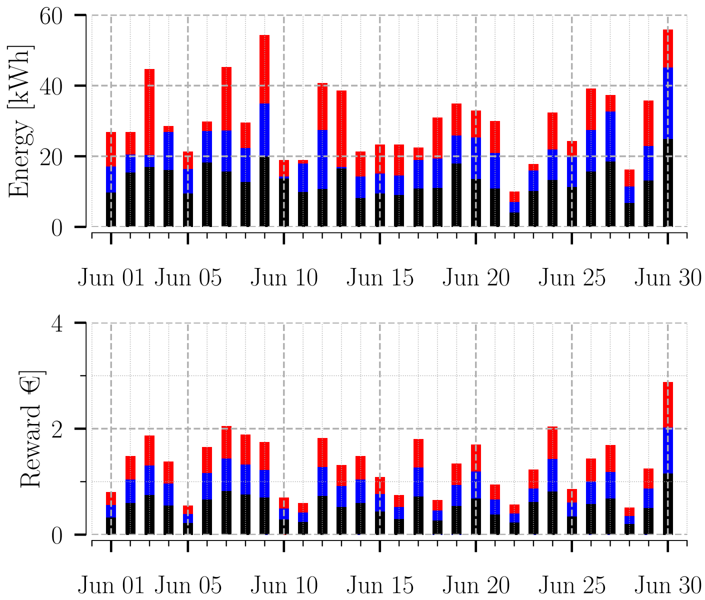

Figure 4. Toy example: [**Top**] cumulative energy sold to the grid within DR periods for entity #1 (red), #2 (blue) and #3 (black). [**Bottom**] Extra profit obtained by entity #1 (red), #2 (blue) and #3 (black).

4.2. Realistic Example

We then considered a realistic REC comprising 30 PV prosumers with BESS, of which five are pure producers (i.e., with zero load). For simplicity, all entities are referred to as prosumers. The peak PV generation capacity $\overline{E}_u$ and BESS capacity $\overline{S}_u$ of each prosumer are reported in Table 2.

For the first day of simulation, the total load and generation profiles of REC members are shown in Figure 5 (top). The net energy E^n injected into the grid is illustrated in Figure 5 (middle), while the overall energy stored in the batteries is depicted in Figure 5 (bottom). The energy selling price $\pi_u^g(t)$ is represented by the green time series in Figure 5 (middle). From the latter figures, it is apparent that during DR periods, entities strategically discharged their storage systems to inject additional energy into the grid. As a result, the prosumers achieved higher profits compared to their baseline scenario, as can be seen in Figure 6 (top), where the daily cumulative profit of all prosumers J^* is compared with that related to standalone operation J_0^*. Figure 6 (bottom) shows the total community reward along with the maximum achievable one, i.e., $\sum_{j=1}^{R} \bar{\gamma}_j$, for all simulation days. The net energy injected into the grid during DR requests is reported in Figure 7, for all days. Note that negative values mean that energy is drawn from the grid during DR requests.

Table 2. Values of parameters for each entity.

Entity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$\overline{E}_u$ [kWh]	29.2	46.4	34.6	19.5	12.8	28.8	62.4	27	26.4	26.8
$\overline{S}_u$ [kWh]	44	120	72	30	20	58	140	56	44	56
Entity	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
$\overline{E}_u$ [kWh]	36	54.4	41.6	13.7	10	52.8	14.4	31.2	74	28.8
$\overline{S}_u$ [kWh]	70	80	60	8	12	88	24	52	92	48
Entity	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
$\overline{E}_u$ [kWh]	25.8	31.2	40.8	12	45	36	24	59.8	35.2	42.8
$\overline{S}_u$ [kWh]	68	52	15	10	60	100	40	108	150	96

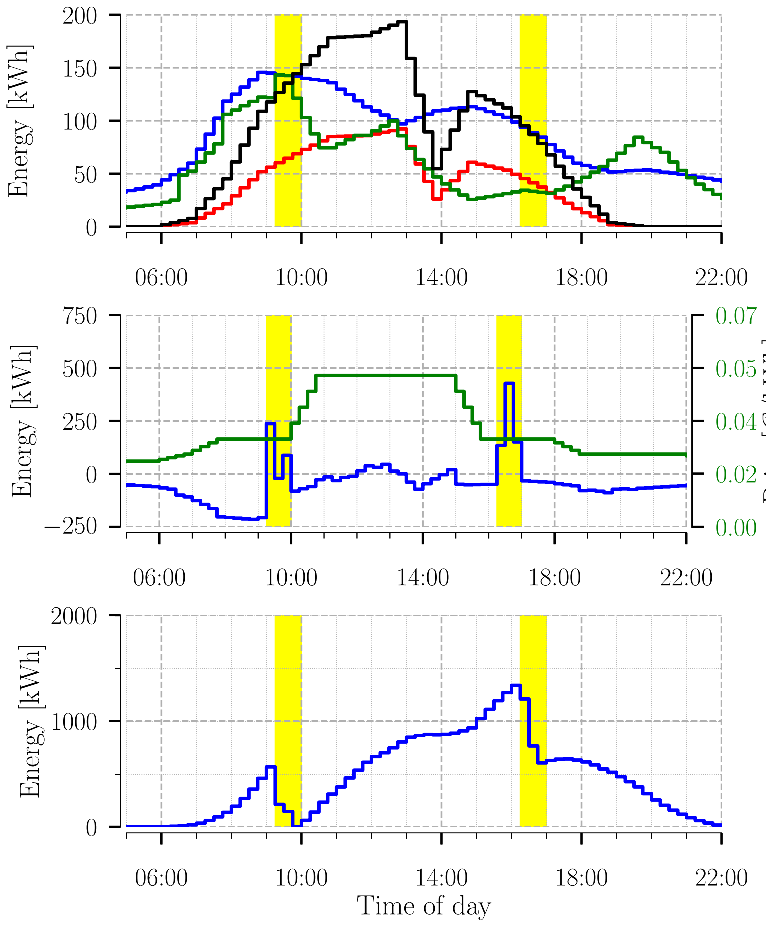

Figure 5. Realistic example: [**Top**] overall load of non-scheduled entities $E^l(t)$ (blue), overall generation of non-scheduled entities $E^p(t)$ (red), overall load of entities with BESS (green), overall generation of entities with BESS (black). [**Middle**] Net energy $E^n(t)$ injected into the grid at REC level (blue) and energy selling price $\pi_u^g(t)$ (green). [**Bottom**] Total energy stored in the BESS by REC entities. Yellow bars denote the time periods of DR requests.

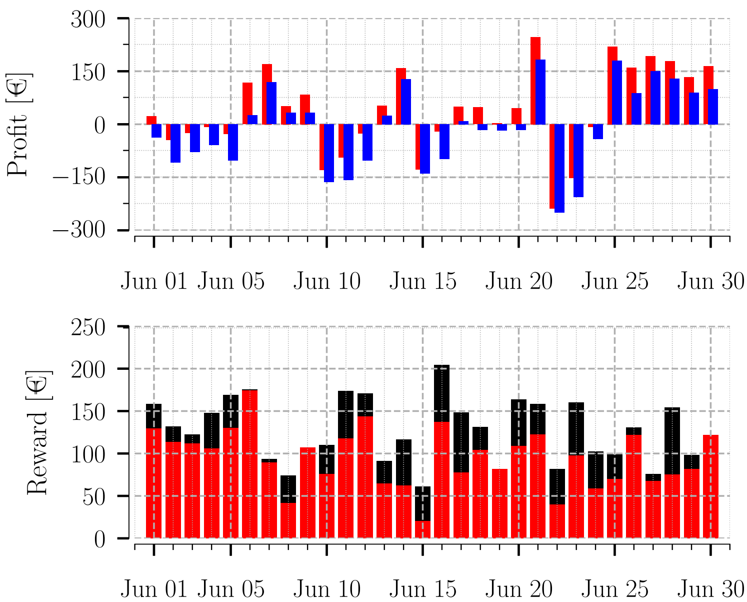

Figure 6. Realistic example: [**Top**] cumulative daily profit of all entities operating independently J_0^* (blue) and joining the REC J^* (red). [**Bottom**] Daily DR reward at REC level (red), and maximum achievable reward (black).

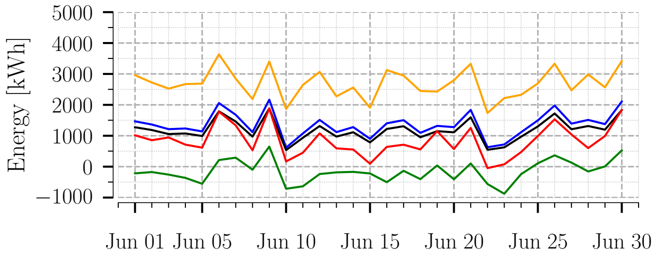

Figure 7. Realistic example: cumulative daily net energy injected into the grid within DR periods $\sum_{j=1}^{R} E_j^{DR}$ (red). Cumulative daily energy thresholds of DR requests $\sum_{j=1}^{R} E_{j,0}^{DR}$ (green), $\sum_{j=1}^{R} E_{j,1}^{DR}$ (black), $\sum_{j=1}^{R} E_{j,2}^{DR}$ (blue), and $\sum_{j=1}^{R} E_{j,3}^{DR}$ (orange).

The red and blue curves in each plot of Figure 8 represent the time series of energy charged ($E_u^c(t)$) and discharged ($E_u^d(t)$), respectively, for BESS units #1 (top), #10 (middle), and #20 (bottom), in the first day of simulation. It is worth noting that, despite a broad range of PV variability, demand patterns, and dynamic price signals, the proposed optimal BESS operation avoided simultaneous storage charging/discharging, i.e., the red and blue curves were never both nonzero, as ensured by Theorem 1.

From a computational viewpoint, the simulation results show that the proposed procedure was efficient even when handling a large number of prosumers. Specifically, for $R = 2$ (10 binary variables), solving 30 instances of Problem 1 (LP) alongside a single instance of Problem 2 (MILP) for an entire day ($T = 96$) required approximately 0.29 s on average. (The proposed procedure was implemented in Python 3 and solved using CPLEX 12 on an Intel i7-11700@3.60 GHz processor with 16 GB RAM. For a case with $R = 4$ (20 binary variables), the average solution time was 0.87 s per day).

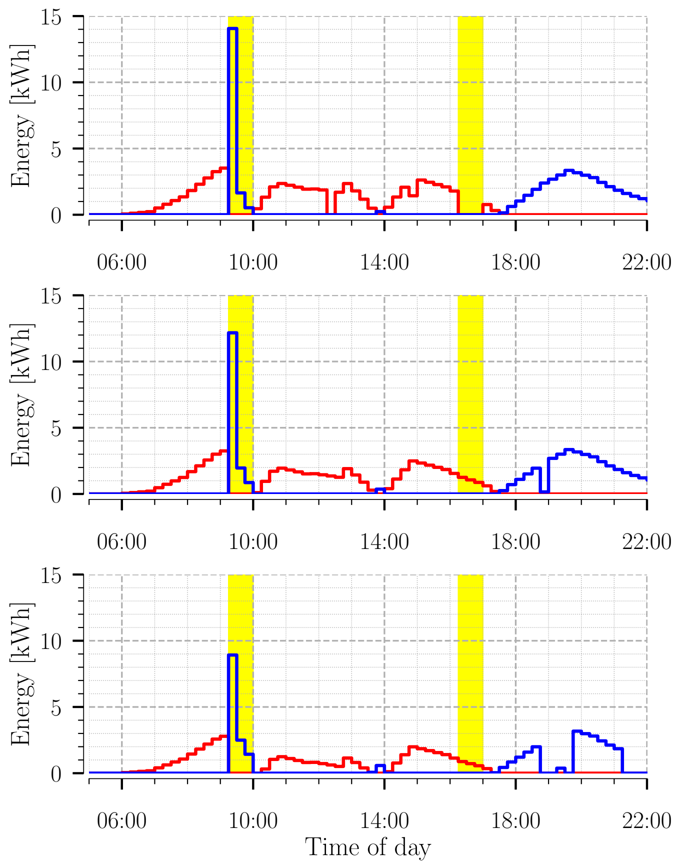

Figure 8. Realistic example: Energy charged (red) and discharged (blue) for entity #1 (**top**), #10 (**Middle**) and #20 (**bottom**), for a selected day. Yellow bars denote the time periods of DR requests.

5. Conclusions

A procedure for the optimal management of the energy storage systems of prosumers joining an REC under DR programs has been proposed. Such a procedure guarantees each entity a total profit no less than that obtained by operating optimally outside the REC. This method requires the solution of a linear program for each schedulable entity and of one low-complexity MILP, thus guaranteeing an affordable computational burden, even for large communities.

Future work will focus on the management of uncertainty in load and generation profiles, exploring complementary approaches such as probabilistic models or robust optimization, as well as incorporating nonlinear pricing structures to enhance applicability in diverse market conditions, such as those with real-time pricing. Moreover, the proposed approach will be specialized to a receding-horizon implementation accounting for real-time pricing schemes. Future directions will also address the introduction of other kinds of entities, such as electric vehicles or shared storage systems, and the extension of the proposed framework to manage multiple energy carriers, such as electricity and heat, jointly dispatched within a unified framework, as well as different energy demand models.

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