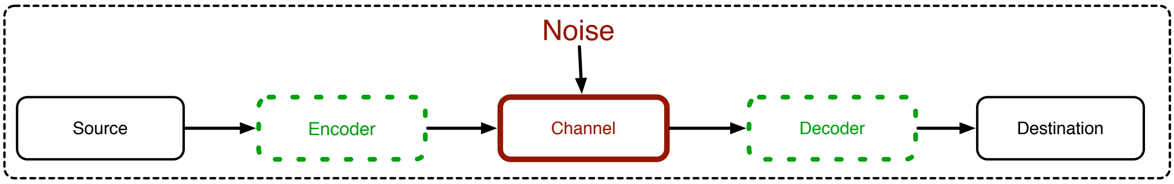
# Detection example 1: digital communications

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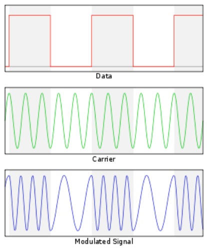
$$0 \leftrightarrow s_0(t) = \sin(\omega_0 t)$$
$$1 \leftrightarrow s_1(t) = \sin(\omega_1 t)$$




$$r(t) = \begin{cases} s_0(t) + n(t) \text{ if '0' sent} \\ s_1(t) + n(t) \text{ if '1' sent} \end{cases}$$

***Detect?***

# Detection example 2: Radar communication

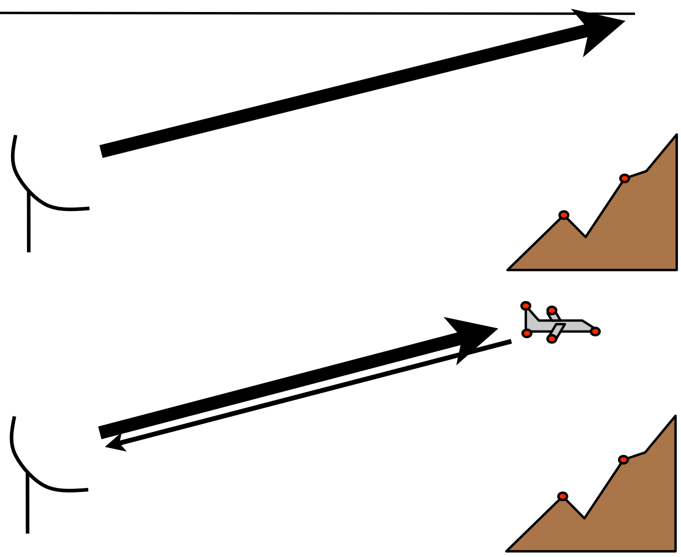
**Send** $s(t) = \sin(\omega_c t), 0 \leq t \leq T$

**Receive**

Hypothesis $\mathcal{H}_0$

$r(t) = n(t),\ 0 \leq t \leq T$

***Detect?***



Hypothesis $\mathcal{H}_1$

$r(t) = V_r \sin((\omega_c + \omega_d)(t - \tau) + \theta_r) + n(t),\ \tau \leq t \leq T + \tau$

## Further examples

- Sonar: enemy submarine!
- Image processing: detect and aircraft from infrared images
- Biomedicine: cardiac arryhthmia from heartbeat sound wave
- Control: detect occurrence of abrupt change in system to be controlled
- Seismology: detect presence of oil deposit

## Difference between detection and estimation?

- Detection:

Discrete set of hypotheses

Right or wrong

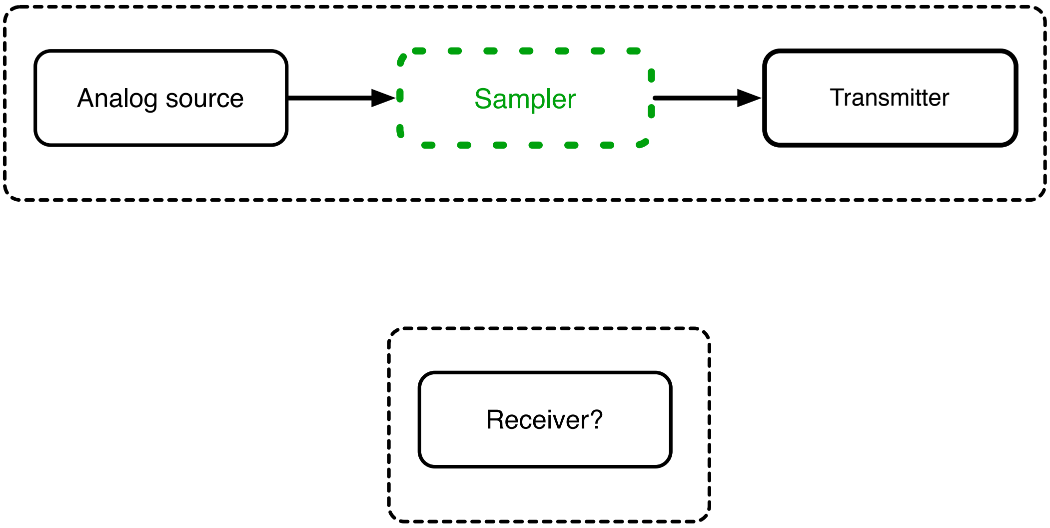
- Estimation:

Continuous set of hypotheses

Almost always wrong - minimize error instead

# Estimation example 1: communications

- Pulse amplitude modulation (PAM)



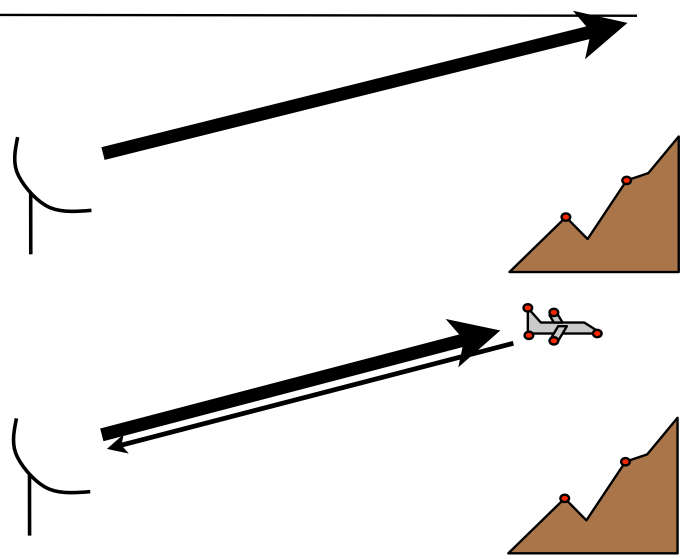

# Estimation example 2: Radar

**Send** $s(t) = \sin(\omega_c t), 0 \leq t \leq T$

**Receive**

Hypothesis $\mathcal{H}_0$

$r(t) = n(t),\ 0 \leq t \leq T$



***Estimate?***

Hypothesis $\mathcal{H}_1$

$r(t) = V_r \sin((\omega_c + \omega_d)(t - \tau) + \theta_r) + n(t),\ \tau \leq t \leq t + \tau$

## Our methods

- Will treat everything generally, with a unified mathematical representation
- Bias towards Gaussian noise
- Examples mainly drawn from communications / radar

## Aside: "Classical" vs. "Bayesian"

### Classical

- Hypotheses/parameters are **fixed, non-random**

### Bayesian

- Hypotheses/parameters are **treated as random variables with assumed priors (or a priori distributions)**

# Outline

*Fundamentals of Statistical Signal Processing, Volume 2: Detection Theory,* by Steven M. Kay, Prentice Hall 1998.

Statistical Detection Theory, Ch.3
Deterministic Signals, Ch.4
Random Signals, Ch.5
Statistical Detection Theory 2, Ch.6
Non-parametric and robust detection

# Estimation: General Minimum Variance Unbiased Estimation

- Bias: (expected value of estimator - true value of data)

$$\text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta = E[\hat{\theta} - \theta]$$

- MVUE:

Unbiased estimator of minimum variance

Always exist?

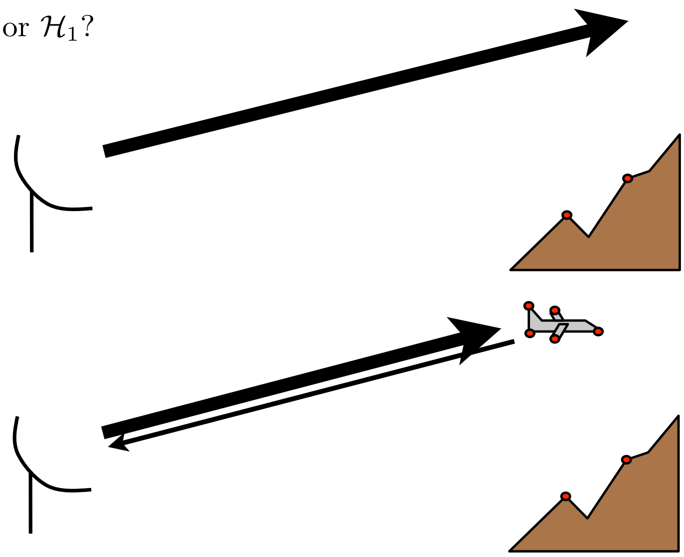
# Detection: Statistical Detection Theory

- Binary hypothesis testing

$\mathcal{H}_0 \text{ or } \mathcal{H}_1?$

Hypothesis $\mathcal{H}_0$

$r(t) = n(t),\ 0 \leq t \leq T$



Hypothesis $\mathcal{H}_1$

$r(t) = V_r \sin((\omega_c + \omega_d)(t - \tau) + \theta_r) + n(t),\ \tau \leq t \leq t + \tau$

# Detection: Statistical Detection Theory

- Binary hypothesis testing

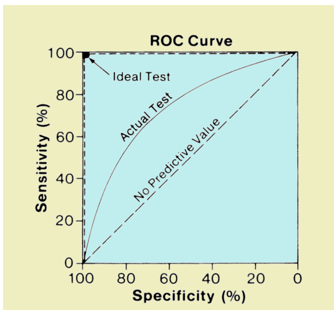
$$\mathcal{H}_0 \text{ or } \mathcal{H}_1?$$

- $P(\mathcal{H}_0; \mathcal{H}_0)$ = prob(decide $\mathcal{H}_0$ when $\mathcal{H}_0$ is true) = prob of correct non-detection
- $P(\mathcal{H}_0; \mathcal{H}_1)$ = prob(decide $\mathcal{H}_0$ when $\mathcal{H}_1$ is true) = prob of missed detection $:= P_M$
- $P(\mathcal{H}_1; \mathcal{H}_0)$ = prob(decide $\mathcal{H}_1$ when $\mathcal{H}_0$ is true) = prob of false alarm $:= P_{FA}$
- $P(\mathcal{H}_1; \mathcal{H}_1)$ = prob(decide $\mathcal{H}_1$ when $\mathcal{H}_1$ is true) = prob of detection $:= P_D$

# Detection: Statistical Detection Theory

Neyman-Pearson (NP): maximize $P_D$ subject to a desired fixed $P_{FA}$.

Receiver Operating Characteristics (ROC) curves




Generalized Bayesian risk which includes as special cases

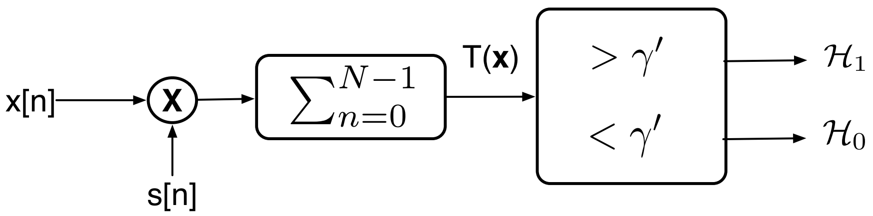
- Minimum probability of error ($\min P_E$) or maximum a posteriori (MAP): $C_{ii} = 0, C_{ij} = 1$ for $i \neq j$.
- Maximum likelihood (ML): $C_{ij} = 0, C_{ij} = 1$ for $i \neq j$ AND all priors are equal, i.e. $P(\mathcal{H}_i) = P(\mathcal{H}_j)$, $\forall i, j$.

# Detection: Deterministic Signals

- How to detect known signals in noise?

$$\mathcal{H}_0 : x[n] = w[n]$$
$$\mathcal{H}_1 : x[n] = s[n] + w[n],$$

- The famous matched filter!




- Generalized matched filter
- > 2 hypotheses

# Detection: Random Signals

- What if s[n] is random?

$$\mathcal{H}_0 : x[n] = w[n]$$
$$\mathcal{H}_1 : x[n] = s[n] + w[n],$$

- Key idea behind *estimator-correlator:*

Estimate the signal first, then matched-filter the estimate

- Linear model simplifies things again...

# Detection: Statistical Decision Theory II

- model for the pdfs under 2 hypotheses are unknown

$$
\begin{aligned}
\mathcal{H}_0 &: x[n] = w[n] \\
\mathcal{H}_1 &: x[n] = s[n] + w[n],
\end{aligned}
$$

- Uniformly most powerful test
- Generalized likelihood ratio test
- Bayesian approach
- Wald test
- Rao test