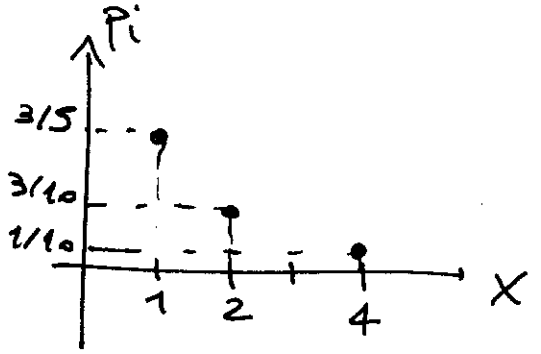
Corso di Statistica – Master E2C

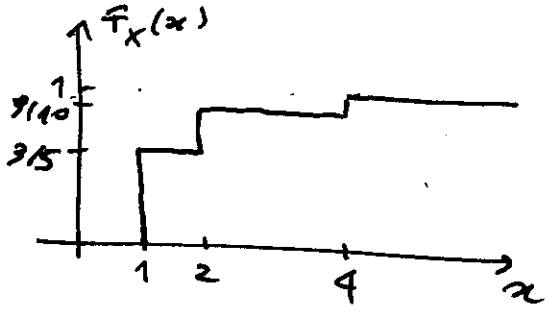
Prova del 15.7.2002

Esercizio 1

a) $$p_i = \begin{cases} 3/5 & X=1 \\ 3/10 & X=2 \\ 1/10 & X=4 \end{cases}$$



b) $$F_X(x) = \begin{cases} 0 & x<1 \\ 3/5 & 1\le x<2 \\ 9/10 & 2\le x<4 \\ 1 & x\ge 4 \end{cases}$$



c) $$m_X = 1\cdot\frac{3}{5} + 2\cdot\frac{3}{10} + 4\cdot\frac{1}{10} = \frac{8}{5}$$

$$E\left[(X-m_X)^2\right] = \left(\frac{6}{10}\right)^2\cdot\frac{3}{5} + \left(\frac{4}{10}\right)^2\cdot\frac{3}{10} + \left(\frac{24}{10}\right)^2\cdot\frac{1}{10} = \frac{840}{1000}$$

d) $$p_i(z) = \begin{cases} 15/45 & z=2 \\ 18/45 & z=3 \\ 3/45 & z=4 \\ 6/45 & z=5 \\ 3/45 & z=6 \end{cases}$$

$$F_2(z) = \begin{cases} 0 & z<2 \\ 15/45 & 2\le z<3 \\ 33/45 & 3\le z<4 \\ 36/45 & 4\le z<5 \\ 42/45 & 5\le z<6 \\ 1 & z\ge 6 \end{cases}$$

e) $$f_{X_3|Z} = \begin{cases} 3/4 & X_3=1 \\ 1/4 & X_3=2 \end{cases}$$

$$P\{X_3=2 \mid Z=6\} = \frac{1}{4}$$

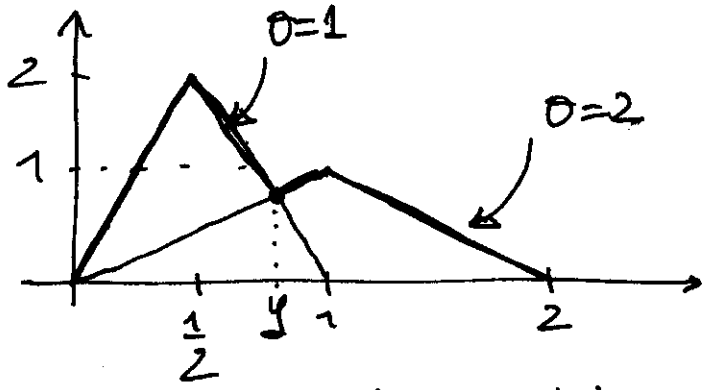
$$E[X_3|Z=6] = \frac{3}{4}\cdot 1 + \frac{1}{4}\cdot 2 = \frac{5}{4}$$

Esercizio 2

a) Dalla figura si vede che $f^{\theta}(x) \geq 0 \quad \forall x$.

Inoltre $\int_{-\infty}^{+\infty} f^{\theta}(x)\,dx = \text{Area Triangolo} = \frac{1}{2}\theta \cdot \frac{2}{\theta} = 1$

b)



Calcolo del punto di intersezione y:

$$\frac{4}{1^2}(1-y) = \frac{4}{2^2}y \quad \rightarrow \quad y = \frac{4}{5}$$

Quindi

$$L(\theta|x) = \begin{cases} f^1(x) & 0 \leq x \leq \frac{4}{5} \\ f^2(x) & \frac{4}{5} < x \leq 2 \end{cases}$$

e

$$\hat{\theta}_{MV} = \begin{cases} 1 & \text{se } 0 \leq x \leq \frac{4}{5} \\ 2 & \text{se } \frac{4}{5} < x \leq 2 \end{cases}$$

c) $$E[X_i] = \int_0^{\frac{\theta}{2}} \frac{4}{\theta^2} x^2\,dx + \int_{\frac{\theta}{2}}^{\theta} \frac{4}{\theta^2}(\theta - x)x\,dx =$$

$$= \frac{4}{3}\cdot\frac{\theta}{8} + \frac{4}{\theta^2}\left[\theta\frac{x^2}{2} - \frac{x^3}{3}\right]_{\frac{\theta}{2}}^{\theta} = \frac{1}{6}\theta + \frac{1}{3}\theta = \frac{1}{2}\theta$$

$$E[\hat{\theta}] = \frac{2}{n}\cdot\sum_{i=1}^{n} E[X_i] = \frac{2}{n}\cdot n \cdot \frac{1}{2}\theta = \theta$$

→ Stima non polarizzata

Esercizio 3

a)

$$E[y_1] = \theta \quad E[y_2] = 2\theta \quad E[y_3] = \theta$$

$$E[\hat{\theta}] = \frac{4}{3}\theta \rightarrow \text{stima polarizzata}$$

b)

$$\hat{\theta}_{LS} = \left(\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} =$$

$$= \frac{1}{6} y_1 + \frac{1}{3} y_2 + \frac{1}{6} y_2$$

$$E[\hat{\theta}_{LS}] = \frac{1}{6}\theta + \frac{1}{3} 2\theta + \frac{1}{6}\theta = \theta \quad \text{stima corretta}$$

c)

$$\hat{\theta}_{GM} = \left(\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}^{-1} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} =$$

$$= \frac{1}{9} y_1 + \frac{2}{9} y_2 + \frac{4}{9} y_3$$

$$E[\hat{\theta}_{GM}] = \frac{1}{9}\theta + \frac{2}{9} \cdot 2\theta + \frac{4}{9}\theta = \theta \quad \text{stima corretta}$$

d)

$$E[(\hat{\theta}_{LS} - \theta)^2] = E\left[\left(\frac{1}{6} v_1 + \frac{1}{3} v_2 + \frac{1}{6} v_3\right)^2\right] =$$

$$= \frac{1}{36}\left(1 + 4 + \frac{1}{4}\right) = \frac{21}{144}$$

$$E[(\hat{\theta}_{GM} - \theta)^2] = E\left[\left(\frac{1}{9} v_1 + \frac{2}{9} v_2 + \frac{4}{9} v_3\right)^2\right] =$$

$$= \frac{1}{81}(1 + 4 + 4) = \frac{1}{9}$$