System Identification and Data Analysis
Lab session on Time Series Prediction

Consider the ARMA process,

$$y(t) - 1.8\,y(t-1) + 0.8075\,y(t-2) = e(t) + 0.5\,e(t-1)$$

where $e(t)$ is a Gaussian white process with zero mean and variance equal to 1.

1. Compute the covariance function $R_y(\tau)$ and plot it for $|\tau| \leq 20$.

2. Simulate a realization of process $y(t)$, for $t = 1, \dots, N$, with $N = 100$.

3. By using the realization obtained at point 2 and the Matlab function `xcorr`, estimate the sample covariance

$$\hat{R}_y(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} (y(t+\tau) - \bar{y})(y(t) - \bar{y})$$

   where $\bar{y} = \frac{1}{N}\sum_{t=1}^{N} y(t)$ is the sample mean. Compare the obtained estimate with the true covariance computed at point 1. Repeat for longer realizations (e.g., $N = 100000$).

4. By using the Matlab function `predict`, compute the 1-step-ahead predictor $\hat{y}(t+1|t)$ and the corresponding sample mean square error

$$\frac{1}{N} \sum_{t=1}^{N} \left\{ y(t+1) - \hat{y}(t+1|t) \right\}^2 .$$

   Compare the true time series $y(t)$ with the predicted one. Estimate the MSE from data and compare it with the theoretical one. Repeat for longer prediction horizons (e.g., $\hat{y}(t+3|t)$) and with realizations of different lengths. Observe the most significant differences.

5. Repeat the exercise for the ARMA process

$$y(t) - 1.8\,y(t-1) + 0.8075\,y(t-2) = e(t) + 2\,e(t-1)$$

   with $e(t)$ as above. Use the Matlab function `spectralfact` to compute the canonical spectral factor of $y(t)$. Compare the predictor and MSE with those obtained in item 4: what is the difference?