System Identification and Data Analysis
Lab session on Nonlinear State Estimation

A mobile robot moves in a 2D environment, represented by the $x - y$ reference frame in Figure 1.

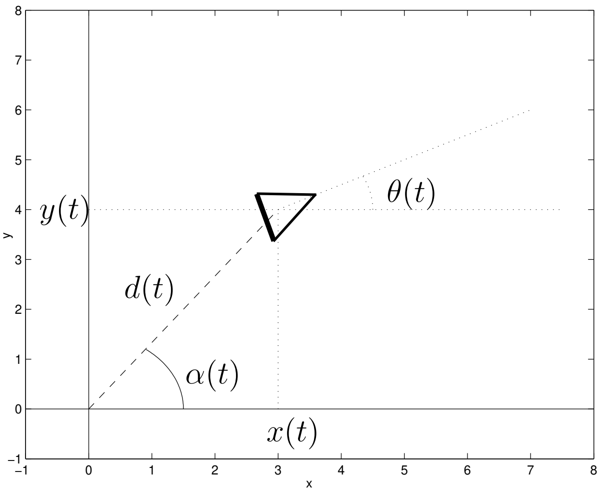

Figura 1:

The robot motion is described by the discrete-time dynamic model

$$
\begin{aligned}
x(t+1) &= x(t) + T_s u_r(t) \cos\theta(t) + w_1(t) \\
y(t+1) &= y(t) + T_s u_r(t) \sin\theta(t) + w_2(t) \\
\theta(t+1) &= \theta(t) + T_s u_a(t) + w_3(t)
\end{aligned}
$$

where

- $x(t), y(t)$ are the coordinates of the center of the robot at time t (expressed in meters);

- $\theta(t)$ is the orientation of the robot, in counterclockwise sense with respect to the x axis (expressed in radiants);

- $u_r(t)$ is the tangential velocity of the robot at time t;

- $u_a(t)$ is the angular velocity of the robot at time t;

- T_s is the sampling time;

- $w(t) = (w_1(t), w_2(t), w_3(t))'$ is the disturbance process, modelled as a white process, with zero mean and covariance matrix Q.

In the origin of the reference frame ($x = 0$, $y = 0$), there is a radar providing distance and angular measurements with respect to the robot position (see Figure 1). Therefore, at every time instant $t = 0, \ldots, N$, the following measurements are available

$$\begin{aligned} d(t) &= \sqrt{x^2(t) + y^2(t)} + v_1(t) \\ \alpha(t) &= \texttt{atan2}(y(t), x(t)) + v_2(t) \end{aligned}$$

where $v(t) = (v_1(t), v_2(t))'$ is the measurement noise (modelled as a white process with zero mean and covariance matrix R), and `atan2` is the four-quadrant inverse tangent, defined as

$$\texttt{atan2}(b, a) = \begin{cases} \arctan(b/a) & \text{if } a > 0 \\ \pi/2 & \text{if } a = 0,\ b > 0 \\ -\pi/2 & \text{if } a = 0,\ b < 0 \\ \arctan(b/a) + \pi & \text{if } a < 0,\ b \geq 0 \\ \arctan(b/a) - \pi & \text{if } a < 0,\ b < 0 \end{cases}$$

The file `data1_labsession_nse.mat` contains:

- the matrix `U` $\in \mathbb{R}^{(N+1)\times 2}$, containing the input sequences $u_r(t)$ (first column) and $u_a(t)$ (second column), for $t = 0, 1, \ldots, N$;

- the matrix `Y` $\in \mathbb{R}^{(N+1)\times 2}$, containing the measurements sequences $d(t)$ (first column) and $\alpha(t)$ (second column), for $t = 0, \ldots, N$;

- the covariance matrices `Q` and `R`;

- the sampling time `Ts` and the vector of times `t`;

- the matrix `X` $\in \mathbb{R}^{(N+1)\times 3}$, containing the sequences of the true state variables $x(t)$ (first column), $y(t)$ (second column) and $\theta(t)$ (third column), for $t = 0, \ldots, N$ [Obviously the true state must be used only for the comparisons and not in the estimation algorithm!!].

(I) By using the motion model and the radar measurements, design and implement an Extended Kalman Filter providing an estimate of the trajectory of the robot and of its orientation.

(II) Compare the estimated trajectory with that obtained by using only the radar measurements and with the true trajectory. Compare the estimated orientation with the true one (be careful with angle wrapping).

(III) Plot the true squared state estimation error and compare it to the corresponding estimate provided by the EKF (the trace of the covariance matrix P). For each state variable, compare the estimation error with the estimated confidence interval $\pm 3\sqrt{P_{ii}(k|k)}$.

(IV) Discuss the role of the initial conditions of the EKF and of the covariance matrices Q ed R, by examining the behavior of the filter when their values change significantly.

(V) Repeat the exercise by using the data contained in the file `data2_labsession_nse.mat`.