Set Membership Pose Estimation of Mobile Robots Based on Angle Measurements

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Abstract

This paper addresses the problem of estimating position and orientation of a mobile robot navigating in an environment for which a landmark-based map is available. A set theoretic approach to the problem is proposed. Estimates of robot position and heading are derived in terms of uncertainty regions, under the hypothesis that the errors affecting all sensors measurements are unknown but bounded. A recursive estimation procedure for localization based on angle measurements is presented. Simulation and experimental results prove the effectiveness of the proposed approach.

1 Introduction

Sensor-based self-localization of a mobile robot is a crucial problem in autonomous long range navigation [1]. The ability to precisely determine its own position and orientation (in the following, we use the term *pose* to refer to both of them) is necessary to perform many fundamental tasks, such as mission accomplishment, or data collection. The localization problem is usually tackled by integrating measurements from proprioceptive sensors (providing information on the robot motion) and exteroceptive sensor (performing measurements on the surrounding environment). Information on robot displacement can be obtained by integrating the outputs of proprioceptive sensors. However, even small noises and sensor drifts lead to error accumulation, causing erroneous estimates [2]. Exteroceptive sensors (e.g. cameras, GPS receivers and laser scanners) provide measurements of geometric relations between the vehicle and some features (described by a suitable map) of the environment. In this case, robot pose can be estimated from scratch, i.e. without having any estimate on the vehicle initial location. The main problem with exteroceptive sensors is uncertainty, due to measurement errors, feature misidentification or map errors. By combining measurements from all sensors, the uncertainty on the exteroceptive measurements and the drift on proprioceptive sensors can be reduced. Depending on the sensors used, the environment representation and the errors model, the problem can be tackled in different ways. Kalman filtering [3], Markov localization techniques [4] or iconic matching [5] have been used with success, providing recursive solutions to the localization problem and a measure of their uncertainty. However, most techniques require statistical assumptions on the nature of the errors.

The contribution of this paper is to propose a solution of the pose estimation problem in the presence of angle measurements affected by bounded errors (for landmark-based localization exploiting angle measurements, see e.g. [6, 7]). No statistical assumption is made on the errors: the only hypothesis is that they are bounded in norm by some quantity. A Set Membership (SM) approach is taken and estimates are given in terms of guaranteed uncertainty regions, containing all *feasible poses* [8, 9, 10]. Though standard SM estimation theory (see, e.g., [11, 12, 13]) is mostly developed for linear estimation problems, the specific nonlinear structure of the feasible pose sets can be exploited to devise recursive procedures for computing and updating the pose uncertainty regions. The resulting dynamic localization procedure enjoys a low computational complexity and proves to be effective in both simulations and real-world experiments.

The paper is structured as follows. Sect. 2 introduces the set theoretic approach to the localization problem. Sect. 3 presents the SM strategy for pose estimation from angle measurements. Robot heading estimation is treated in Sect. 4, while Sect. 5 provides a refinement procedure for obtaining tighter set estimates. Simulation and experimental results are reported in Sect. 6. Finally, some concluding remarks are drawn in Sect. 7.

2 SM approach to the localization problem

Let us consider a vehicle navigating in a 2D environment, and let $p(k) = [x(k)\ y(k)\ \theta(k)]' \in \mathcal{Q} \triangleq \mathbb{R}^2 \times [-\pi\ \pi]$ be the pose of the agent at time k (where $\theta(k)$ represents the heading of the vehicle w.r.t. the positive x-axis). Under the assumption of slow robot dynamics and if translation and rotation measurements $u(k)$ from odometric sensors are available, the robot dynamics is described by the linear discrete-time model

$$p(k+1) = p(k) + u(k) + G(k)w(k), \qquad (1)$$

where $w(k) \in \mathbb{R}^3$ models the errors affecting measurements $u(k)$ (possibly shaped by a suitable matrix $G(k)$).

Exteroceptive sensors provide measurements on the environment. When the robot moves in a static environment, these measurements depend only on the robot pose $p(k)$ and the landmark positions, that are known. As a consequence, the outputs of the sensors can be described by equations of the form

$$c_i(k) = \mu_i(p(k)) + v_i(k) \qquad i = 1, \ldots, m \qquad (2)$$

where m is the number of measurements performed at time k, $\mu_i(p)$ is a (nonlinear) function modeling the i-th measurement process and $v_i(k)$ is the noise affecting that measurement. Hence, the localization problem can be stated as follows.

Localization Problem: Let $\hat{p}(0)$ be an estimate of the initial robot pose. Given the dynamic model (1) and the measurement equation (2), construct an estimator of the vehicle pose $p(k)$ at each time instant $k = 1, 2, \ldots$

This problem can be tackled in different ways, depending on the hypotheses on the unknown disturbances $w(k)$ and $v_i(k)$ in (1) and (2). When statistical assumptions on the errors are considered, the estimate can be computed via the extended Kalman filter, or using Markov localization and Bayes rule. However, real-world uncertainties may include also systematic errors or non-gaussian, non-white noise, whose statistical properties are very difficult to estimate. In this paper a different approach is presented based on the assumption that the disturbances are unknown-but-bounded, i.e.

$$|w_i(k)| \leq \epsilon_i^w(k) \quad i = 1, 2, 3 \qquad (3)$$
$$|v_i(k)| \leq \epsilon_i^v(k) \quad i = 1, \ldots, m \qquad (4)$$

where $\epsilon_i^w(k)$, $\epsilon_i^v(k)$ are known scalars. Assumption (4) allows one to define, for each measurement in (2), a set where the navigator pose is allowed to lie, given by

$$\mathcal{M}_i = \mathcal{C}(c_i, \epsilon_i^v) = \{p \in \mathcal{Q} : c_i - \epsilon_i^v \leq \mu_i(p) \leq c_i + \epsilon_i^v\}. \quad (5)$$

The shape of this set depends on the (nonlinear) function in (2) and can generally be nonconvex, nonconnected and/or unbounded. At every time k, the pose will be constrained to lay into the intersection of m different sets defined by (5), thus giving the *measurement set* $\mathcal{M}(k) = \bigcap_{i=1}^m \mathcal{M}_i(k)$. We point out that, if assumption (4) is verified, the set $\mathcal{M}$ is not empty. On the other hand, an empty intersection implies that at least one of the constraints (4) is violated. The localization problem can be formulated in terms of dynamic estimate of feasible sets.

Set Membership localization problem: Let $P(0) \subset \mathcal{Q}$ be a set containing the initial pose $p(0)$. Find at each time $k = 1, 2, \ldots$ the *feasible pose set* $P(k|k) \subset \mathcal{Q}$ containing all vehicle poses $p(k)$ that are compatible with the dynamics (1), the measurements (2) collected up to time k and assumptions (3)-(4).

The solution of this problem is obtained from the following recursion, stemming out directly from (1)-(2)

$$P(0|0) = P(0) \qquad (6)$$
$$P(k|k-1) = P(k-1|k-1) + u(k-1) + G(k-1)\mathrm{Diag}[\epsilon^w(k-1)]\mathcal{B}_\infty \qquad (7)$$
$$P(k|k) = P(k|k-1) \cap \mathcal{M}(k) \qquad (8)$$

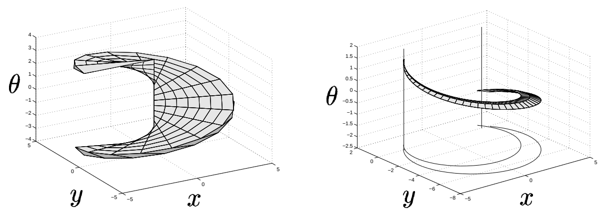
where $\mathcal{B}_\infty$ is the unit ball in the ℓ_∞ norm, defined as $\|v\|_\infty = \max_i |v_i|$, and $\mathrm{Diag}[v]$ denotes the diagonal matrix with vector v on the diagonal. Note that algebraic operators in (6)-(8) are to be intended as set operators. Equation (7) is based on the information provided by proprioceptive sensors. Due to the noise affecting odometers, uncertainty will grow during this step. Equation (8) exploits exteroceptive measurements to reduce uncertainty on the robot position. It requires the computation of the intersections of highly structured nonlinear sets in $\mathbb{R}^3$. The computational complexity of such intersections quickly becomes too high to be tractable (see, e.g. [9, 10]) and approximation techniques performing a tradeoff between complexity and accuracy, are usually pursued. Set membership estimation theory provides efficient approximation algorithms for linear relations like (7) [14]. Problems introduced by nonlinear equations such as (8) depend on the particular form of the measurement equation (2), i.e. on the exteroceptive sensors available to the robot. Two different approximations are introduced in the evaluation of (7)-(8).

• **Set bounding**: the actual sets in (7)-(8), are replaced by approximating regions belonging to a class of fixed, simpler structure. In order to obtain sets containing the true robot pose, the smallest outer approximation in the chosen class is sought. Given a class of regions of fixed structure $\mathcal{R}$, and denoted by $\overline{\mathcal{R}}\{\mathcal{S}\}$ the minimum volume region in $\mathcal{R}$ containing the set $\mathcal{S}$, the desired approximation of the recursion (6)-(8) is given by

$$\mathcal{R}(0|0) = \overline{\mathcal{R}}\{P(0)\} \qquad (9)$$
$$\mathcal{R}(k|k-1) = \overline{\mathcal{R}}\{\mathcal{R}(k-1|k-1) + u(k-1) + G(k-1)\mathrm{Diag}[\epsilon^w(k-1)]\mathcal{B}_\infty\} \qquad (10)$$
$$\mathcal{R}(k|k) = \overline{\mathcal{R}}\{\mathcal{R}(k|k-1) \cap \mathcal{M}(k)\}. \qquad (11)$$

• **State decomposition**: evaluation of the actual feasible sets in the entire state space is replaced by that of smaller dimension subsets within suitable subspaces. Thus simpler sets are obtained. Set intersections are then performed in these subspaces, and afterwards the overall feasible pose set is approximated by the Cartesian product of the evaluated subsets. This strategy introduces an approximation in the sense that correlation between state elements belonging to different projected subspaces is lost. We point out that the set obtained by the final product always contains the real set.

In the next section we will present an efficient algorithm for the case of angle measurements, performing both set bounding and state decomposition. Position



a b

Figure 1: a) Measurement set $\mathcal{M}_i$ associated to a relative orientation measurement; b) Intersection between two distinct measurement sets and its projection on the (x, y) plane.

and orientation will be estimated separately, and their feasible subsets will be evaluated using simple outer approximating regions.

3 Set Membership localization using angle measurements

Let us suppose that the robot is navigating in an environment described by a 2D map containing landmarks (i.e., there are isolated features that can be detected). Let the coordinates (x_{L_i}, y_{L_i}) of N landmarks be available to the robot. For each landmark L_i, the navigator can measure the angle between its orientation and the line connecting it to the landmark, i.e.

$$\mu_i(p(k)) = \text{atan}_2(\Delta y_i(k), \Delta x_i(k)) - \theta(k), \qquad (12)$$

where $\Delta y_i(k) = y_{L_i} - y(k)$, $\Delta x_i(k) = x_{L_i} - x(k)$ and atan_2 is the four quadrant inverse tangent. This kind of measurement is available from panoramic cameras or, as a byproduct, from stereo couples or range finders. Equation (12) implies that the measurement set $\mathcal{M}_i$ in (5) has the following expression

$$\mathcal{M}_i = \{p \in \mathcal{Q} : |\text{atan}_2(\Delta y_i, \Delta x_i) - \theta - c_i| \leq \epsilon_i^v\}. \quad (13)$$

This set is a portion of $\mathcal{Q}$ delimited by two helicoids (a part of this set, limited in the (x, y) plane is shown in Fig. 1a). Each set $\mathcal{M}_i$ is unbounded. However, the intersection of two measurement sets $\mathcal{M}_i$, related to distinct landmarks, is always bounded. Moreover, its projection on the (x, y) plane is the well-known “thickened ring” [15] , i.e. the region between two circular arcs with the same extreme points: the two landmarks (see Fig. 1b). Evaluating the difference between two measurement equations (2), with $\mu(\cdot)$ given by (12), one obtains

$$c_i - c_j = \tau_{ij} + v_i - v_j \qquad (14)$$

where τ_{ij} is the visual angle between two distinct landmarks (the angle formed by the rays from the vehicle position to each landmark). The position uncertainty region associated to noisy measurements c_i and c_j is the thickened ring $\mathcal{T}_{ij} = \{[x\ y]' : |c_i - c_j - \text{atan}_2(\Delta y_i, \Delta x_i) + \text{atan}_2(\Delta y_j, \Delta x_j)| < \epsilon_i^v + \epsilon_j^v\}$ (see, e.g. [7, 16]). Given N landmarks, one can intersect up to $N(N-1)/2$ thickened rings, considering all the landmark pairs. This gives the set $\mathcal{T} = \bigcap_{i>j}^{N} \mathcal{T}_{ij}$ which is the best possible set membership position estimate, based on the available measurements.

Since (14) does not depend on the robot orientation, a 2D set $\mathcal{R}_l(k|k)$ containing the feasible vehicle positions, and an interval $\mathcal{R}_o(k|k)$ for the admissible robot orientations can be evaluated separately. Thickened rings are sets bounded by nonlinear curves, so set approximation is needed. In the following, an algorithm that uses boxes as approximating sets is outlined. An axis-aligned *box* is defined by $\mathcal{B} = \mathcal{B}(c, b) = \{q : q = c + \text{Diag}[\text{b}]\alpha, \|\alpha\|_\infty \leq 1\}$ where c is the center of the box and the absolute values of the elements of $2b$ represent the length of the edges. We observe that premultiplication of a box by a (nonsingular) square matrix gives a parallelotope, which is defined as $\mathcal{P} = \mathcal{P}(c, T) = \{q : q = c + T\alpha, \|\alpha\|_\infty \leq 1\}$, where c is the center and T is a (nonsingular) matrix whose column vectors represent the edges of the parallelotope.

Using (10)-(11) and the state decomposition, we have to solve the following approximation problems:

B1. Compute the minimum volume 3D box containing the sum of a box and a parallelotope in $\mathbb{R}^3$ (eq. (10));

B2. Compute the minimum volume 2D box $\mathcal{R}_l(k|k)$ containing the intersection of a box in $\mathbb{R}^2$ with the set $\mathcal{T}$ (projection of (11) on the $x - y$ plane);

B3. Compute an interval $\mathcal{R}_o(k|k)$ containing the feasible orientation set of the vehicle (projection of (11) on the subspace spanned by θ).

Notice that the cartesian product $\mathcal{R}_l(k|k) \otimes \mathcal{R}_o(k|k)$ is the desired approximation of the feasible set $P(k|k)$.

Optimal solution to problems B1, and a suboptimal recursive solution to problem B2 have been presented in [16]. In the next section, a technique for computing interval approximations for the robot orientation (i.e., a suboptimal solution to problem B3) will be introduced.

4 Orientation set approximation

Let $c_i(k)$, $i = 1, \ldots, N$, be the angle measurements at time k and $\mathcal{R}_l(k|k)$ be a box containing all the vehicle positions compatible with the robot dynamics and the measurements up to time k.

¿From each angle measurements $c_i(k)$, relative to a landmark L_i outside the box $\mathcal{R}_l(k|k)$, an interval containing the actual robot orientation can be derived. In Fig. 2, the landmark L_i and the box $\mathcal{R}_l(k|k)$ are shown. Lines s_{i_1} and s_{i_2} connect landmark L_i and the box vertices $\underline{V}$ and $\overline{V}$, chosen so that the angle φ_i with the X axis is respectively minimized ($\underline{\varphi}_i$) and maximized ($\overline{\varphi}_i$). The equations of these lines are known, being the landmark and the vertices coordinates known. The angle θ between the x axis of the robot reference system $X^{(r)}$ and the X axis is unknown, and so are the angles $\underline{\alpha}_i$, $\overline{\alpha}_i$ between $X^{(r)}$ and lines s_{i_1}, s_{i_2}. ¿From a simple

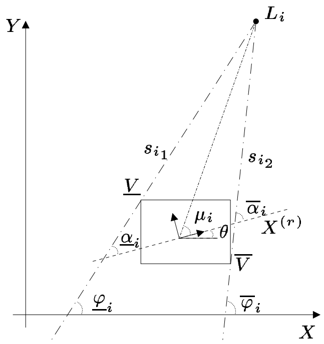


Figure 2: Geometric setting for orientation estimation.

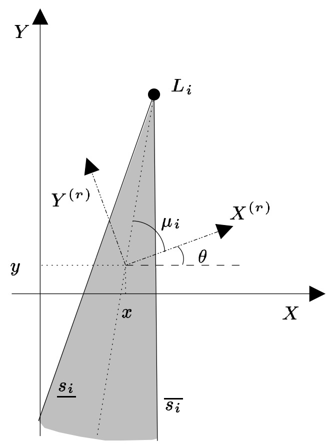


Figure 3: Position estimation refinement: the set $\mathcal{M}_{l_i}(k, \mathcal{R}_o(k|k))$.

geometric reasoning, one has for the visual angle μ_i

$$\underline{\varphi}_i - \theta = \underline{\alpha}_i \leq \mu_i \leq \overline{\alpha}_i = \overline{\varphi}_i - \theta, \tag{15}$$

in which $\underline{\varphi_i}$ and $\overline{\varphi_i}$ are known. Equation (15) can be rewritten as

$$\underline{\varphi}_i - \mu_i \leq \theta \leq \overline{\varphi}_i - \mu_i. \tag{16}$$

Since only a noisy measurement c_i of the visual angle μ_i is available, (16) has to be modified according to (2) and assumption (4) on error bounds, thus giving

$$\underline{\varphi}_i - c_i - \epsilon_i^v \leq \theta \leq \overline{\varphi}_i - c_i + \epsilon_i^v \tag{17}$$

which provides a restriction on the admissible orientation values. Setting $\mathcal{M}_{o_i} = [\underline{\varphi}_i - c_i - \epsilon_i^v,\ \overline{\varphi}_i - c_i + \epsilon_i^v]$, the desired outer approximation of the feasible orientation set is obtained by computing the intersection:

$$\mathcal{M}_o(k, \mathcal{R}_l(k|k)) = \bigcap_{i:\ L_i \notin \mathcal{R}_l(k|k)} \mathcal{M}_{o_i} \tag{18}$$

Notice that this is an approximation of the exact feasible orientation set, because $\mathcal{R}_l(k|k)$ is an approximation of the exact feasible position set.

The overall localization procedure is given by the following algorithm where, $\Pi = [I_2\ 0_{2\times 1}]$ and $\Omega = [0\ 0\ 1]$.

0. Let $\mathcal{R}(0|0) = \overline{\mathcal{R}}\{P(0)\}$.

For $k = 1, 2, \ldots$

1. Find $\mathcal{R}(k|k-1)$ in (10) (computed as in [16]);

2. Find $\mathcal{R}_l(k|k) \supset \overline{\mathcal{R}}\{\Pi\mathcal{R}(k|k-1) \cap \mathcal{T}\}$ as in [16];

3. Let $\mathcal{R}_o(k|k) = \Omega\mathcal{R}(k|k-1) \cap \mathcal{M}_o(k, \mathcal{R}_l(k|k))$;

4. Let $\mathcal{R}(k|k) = \mathcal{R}_l(k|k) \otimes \mathcal{R}_o(k|k)$.

The set $\mathcal{R}(k|k)$ is the desired outer approximation of the admissible pose set, as it satisfies $P(k|k) \subset \mathcal{R}(k|k)$ by construction.

5 Refinement of the approximating sets

In this section it will be shown how, given the interval $\mathcal{R}_o(k|k)$ containing all feasible robot orientations, it is possible to refine the position estimate, by reducing the area of the box $\mathcal{R}_l(k|k)$ containing the feasible position set. In fact, if θ is allowed to vary inside the interval $\mathcal{R}_o(k|k)$, then the robot position must lie in a sector of the $x - y$ plane (the grey region of Fig. 3 is an example). Let $\hat{\theta}(k) = (\underline{\theta} + \overline{\theta})/2$, $a(k) = (\overline{\theta} - \underline{\theta})/2$ where $\underline{\theta} = \min\{\mathcal{R}_o(k|k)\}$ and $\overline{\theta} = \max\{\mathcal{R}_o(k|k)\}$. For each landmark L_i, it is possible to define a set containing the vehicle position as

$$\begin{aligned}&\mathcal{M}_{l_i}(k, \mathcal{R}_o(k|k)) = \{[x(k)\ y(k)]' \in \mathbb{R}^2 : \\ &|c_i + \hat{\theta}(k) - \text{atan}_2(\Delta y_i, \Delta x_i)| \leq \epsilon_i^v(k) + a(k)\}.\end{aligned} \tag{19}$$

It is easy to see that this is a sector delimited by two lines originating in L_i (denoted by $\underline{s_i}$ and $\overline{s_i}$ in Fig. 3). Considering the intersection $\mathcal{M}_l = \bigcap_{i=1}^N \mathcal{M}_{l_i}$ we obtain a new admissible position set. The uncertainty on the robot position can be reduced by evaluating the minimum area box containing the intersection of such a set with $\mathcal{R}_l(k|k)$ (Fig. 4)

$$\mathcal{R}_l^{(i)}(k|k) = \overline{\mathcal{R}}\left\{\mathcal{R}_l^{(i-1)}(k|k) \cap \mathcal{M}_l(k)\right\} \tag{20}$$

where the superscript (i) is the number of refinement iterations performed on the set $\mathcal{R}_l(k|k)$. Once the position uncertainty set $\mathcal{R}_l(k|k)$ has been tightened, also the interval of admissible orientations $\mathcal{R}_o(k|k)$ can be reduced. This is done by repeating step 3 of the algorithm in the previous section, using the position box $\mathcal{R}_l^{(i)}(k|k)$ computed in (20).
By iterating the computation of $\mathcal{R}_l(k|k)$ and $\mathcal{R}_o(k|k)$, one can further reduce the uncertainty affecting position and orientation estimates. This is done by setting

$$\begin{aligned}\mathcal{R}_l^{(0)}(k|k) &= \overline{\mathcal{R}}\{\Pi\mathcal{R}(k|k-1) \cap \mathcal{T}\} \\ \mathcal{R}_o^{(0)}(k|k) &= \Omega\mathcal{R}(k|k-1) \cap \mathcal{M}_o(k, \mathcal{R}_l^{(0)}(k|k))\end{aligned}$$

and then repeating for $i = 1, 2, \ldots$

$$\mathcal{R}_l^{(i)}(k|k) = \overline{\mathcal{R}}\left\{\mathcal{R}_l^{(i-1)}(k|k) \cap \mathcal{M}_l(k, \mathcal{R}_o^{(i-1)}(k|k))\right\} \tag{21}$$

$$\mathcal{R}_o^{(i)}(k|k) = \mathcal{R}_o^{(i-1)}(k|k) \cap \mathcal{M}_o(k, \mathcal{R}_l^{(i)}(k|k)). \tag{22}$$

Solution of (21) requires the computation of the minimum box containing the intersection of a box and N

sectors $\mathcal{M}_{l_i}$. This boils down to 4 linear programming problems with $2N+4$ constraints. In order to obtain less computational demanding algorithms, suboptimal solutions based on recursive set approximation procedures can be adopted [17]. By processing sequentially the intersection of each set $\mathcal{M}_{l_i}$ with the current approximating box, the computational burden reduces to $O(N)$ operations for each set approximation (21). Note that (22) can be easily computed exactly, as it requires the intersection of at most $N+1$ intervals (see (18)).

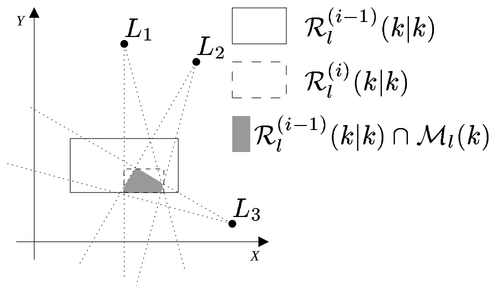


Figure 4: Updating of the admissible position set.

Several heuristic criteria for stopping the above recursion can be formulated. In practice, very few iterations are sufficient to significantly reduce the size of the uncertainty boxes.

6 Experimental testing

6.1 Numerical simulation results

In this section, simulation results of localization experiments are reported. First, the proposed algorithm has been tested in a static setting. At time k, the vehicle is supposed to be located at the center of a square room of 20 meters side, with no information on its orientation. This means that $\mathcal{R}(k|k-1) = \mathcal{B}([0\ \ 0\ \ 0]', [10\ \ 10\ \ \pi]')$, in a reference system centered at the vehicle position. It is assumed that 5 landmarks are identified in the scene. The relative angle measurements are corrupted by additive noise $v_i(k)$, generated as an uniformly distributed signal satisfying (4) with constant bound $\epsilon_i^v(k) = \epsilon^v$, $\forall i$. As nominal estimates of the vehicle position and orientation the center of the box $\mathcal{R}_l(k|k)$ and the center of the interval $\mathcal{R}_o(k|k)$ are considered. Position and orientation errors (in meters and degrees, respectively) and the associated uncertainties, for different values of ϵ^v ranging from 0.5 to 2.5 degrees, are reported in Table 1. Results are averaged over 1000 different landmark configurations in the square room. Table 2 reports the same results for the case of 10 landmarks. In this case errors are remarkably reduced, as expected.

Error bound	Position error (m)	Box area (m^2)	Orientation error (deg)	Orientation width (deg)
0.50	0.07	0.21	0.25	3.07
1.00	0.14	0.48	0.48	5.33
1.50	0.19	1.13	0.72	8.12
2.00	0.26	1.76	0.94	10.36
2.50	0.31	2.25	1.08	12.50

Table 1: Simulation results with 5 landmarks.

Error bound	Position error (m)	Box area (m^2)	Orientation error (deg)	Orientation width (deg)
0.50	0.02	0.0094	0.09	1.00
1.00	0.04	0.039	0.18	1.97
1.50	0.07	0.097	0.29	3.08
2.00	0.08	0.16	0.40	4.17
2.50	0.11	0.22	0.46	4.89

Table 2: Simulation results with 10 landmarks.

6.2 Experimental results

In order to test the proposed algorithm in a real-world dynamic setting, several experiments with the mobile robot Nomad XR4000 have been carried out. This vehicle has an holonomic drive system and is equipped with a SICK LMS 200 laser rangefinder, whose scanning angle width is 180° with a resolution of 0.5°.

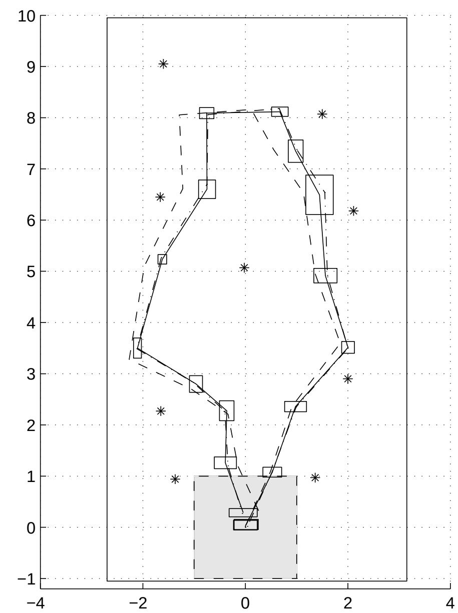


Figure 5: Dynamic localization: landmarks (*), position uncertainty (boxes), nominal trajectory (dashed line), true trajectory (dash-dotted line) and estimated trajectory (solid line).

To simplify landmark extraction from planar range scans, the following scenario has been considered: in a room of approximately 60 m^2, 9 artificial landmarks have been placed in known positions. Figs. 5-6 show the results of a typical experiment. The robot was programmed to follow a predefined piecewise linear trajectory (dashed-line in Fig. 5), relying only on its odometry system; after each movement, data from a 180° range scan were collected. Concerning the motion model, the following assumption have been made: $G(k) = I$ and $\epsilon^w(k) = \epsilon^w = [0.2m\ 0.2m\ 5°]'$. Time-varying measurement error bounds $\epsilon_i^v(k)$ (see equation (4)) have been derived directly as byproducts of the landmark extraction phase. This has been per-

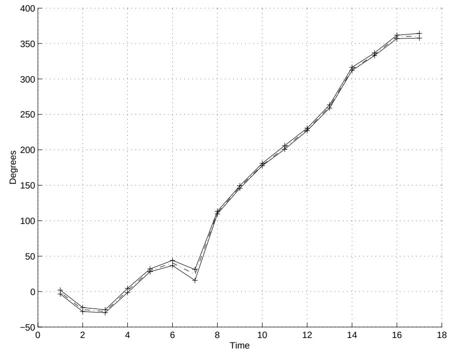


Figure 6: Dynamic localization: orientation uncertainty intervals and true orientation (dashed-dotted line).

formed by properly filtering the signal provided by the rangefinder scan. Each landmark corresponds to a certain angle interval in the filtered signal (clearly dependent on the distance from the robot to the selected landmark), whose half amplitude has been chosen as $\epsilon_i^v(k)$. The average value of $\epsilon_i^v(k)$ turned out to be about $1°$. At time $k = 0$, the uncertainty position set was $\mathcal{R}_l(0|0) = \mathcal{B}([0\ 0]', [1\ 1]')$ (dashed gray box in Fig. 5), while no information was supposed to be available on the initial robot orientation, i.e. $\mathcal{R}_o(0|0) = \mathcal{B}(0, \pi)$. In Fig. 5, it can be noticed that the true trajectory followed by the vehicle (dash-dotted line) rapidly deviated from the nominal one (dashed line), due to the well-known error accumulation associated with dead-reckoning techniques. The nominal trajectory estimated via the proposed set membership technique (solid line, connecting the centers of the position uncertainty boxes) follows quite well the true path. Also orientation is estimated quite precisely (see Fig. 6). On average, nominal position and orientation errors (measured with respect to the center of the corresponding feasible sets) are less than 5 cm and $0.4°$, respectively, while the area of the position uncertainty box is about 0.1 m^2 and the width of the orientation uncertainty interval is $5.4°$. The worst estimation performance, both in terms of nominal error and uncertainty region size, were obtained at time $k = 7$, due to the reduced number of visible landmarks (only 3, while on average, approximatively 6 landmarks were visible). Note that these results are in good agreement with those of numerical simulations presented in Sect. 6.1, the slightly uncertainty increase being due to the presence of odometers errors in the vehicle motion model.

7 Conclusion

A set membership technique for pose estimation of a mobile robot, based on angle measurements with respect to known landmarks, has been presented.
The main features of this approach are the following:
- no assumptions are required on the errors in the robot dynamics model and on the noises affecting sensor measurements, except that they must be bounded;
- the SM localization algorithm provides both nominal estimates of the robot pose and the associated guaranteed uncertainty sets for position and orientation;
- the algorithm is simple and can be implemented in any commercial robot. In this respect, strategies for trading off computational burden and tightness of the approximating regions can be devised (see Sect. 5).

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