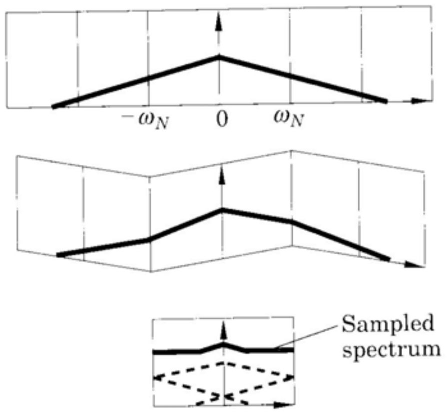

Figure 7.11 Frequency folding.

Frequency Folding

Equation (7.3) can also be given another interpretation. The graph of the spectrum of the continuous-time signal is first drawn on a paper. The paper is then folded at abscissas that are odd multiples of the Nyquist frequency, as indicated in Fig. 7.11. The sampled spectrum is then obtained by adding the contributions, with proper phase, from all sheets.

Prefiltering

A practical difficulty is that real signals do not have Fourier transforms that vanish outside a given frequency band. The high-frequency components may

Table 7.1 Damping ζ and natural frequency ω for Butterworth, ITAE (Integral Time Absolute Error), and Bessel filters. The higher-order filters with arbitrary bandwidth ω_B are obtained by cascading filters of the form (7.12).

	Butterworth		ITAE		Bessel	
Order	ω	ζ	ω	ζ	ω	ζ
2	1	0.71	0.99	0.71	1.27	0.87
4	1	0.38	1.49	0.32	1.60	0.62
		0.92	0.84	0.83	1.43	0.96
6	1	0.26	1.51	0.24	1.90	0.49
		0.71	1.13	0.60	1.69	0.82
		0.97	0.92	0.93	1.61	0.98

Table 7.2 Approximate time delay T_d of Bessel filters of different orders.

Order	T_d
2	$1.3/\omega_B$
4	$2.1/\omega_B$
6	$2.7/\omega_B$

appear to be low-frequency components due to aliasing. The problem larly serious if there are periodic high-frequency components. To avo problem, it is necessary to filter the analog signals before sampling be done in many different ways.

Practically all analog sensors have some kind of filter, but t seldom chosen for a particular control problem. It is therefore ofter to modify the filter so that the signals obtained do not have freque the Nyquist frequency.

Sometimes the simplest solution is to introduce an analog filte the sampler. A standard analog circuit for a second-order filter is

$$G_f(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

Higher-order filters are obtained by cascading first- and secon tems. Examples of filters are given in Table 7.1. The table gives bandwidth $\omega_B = 1$. The filters get bandwidth ω_B by changing the fa to

$$\frac{\omega^2}{(s/\omega_B)^2 + 2\zeta\omega(s/\omega_B) + \omega^2}$$

where ω and ζ are given by Table 7.1. The Bessel filter has a li curve, which means that the shape of the signal is not distorted Bessel filters are therefore common in high-performance systems.

The filter must be taken into account in the design of the the desired crossover frequency is larger than about $\omega_B/10$, where bandwidth of the filter. The Bessel filter can, however, be approxi a time delay, because the filter has linear phase for low frequencie shows the delay for different orders of the filter. Figure 7.12 shows th of a sixth-order Bessel filter and a time delay of $2.7/\omega_B$. This prope that the sampled-data model including the antialiasing filter can b to contain an additional time delay compared to the process. Assur bandwidth of the filter is chosen as

$$|G_f(i\omega_N)| = \beta$$