

Sum-Of-Norms MPC for Linear Periodic Systems with Application to Spacecraft Rendezvous

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Abstract— In this paper, an MPC solution is devised for linear time-periodic optimal regulation problems requiring the minimization of a sum of vector norms. Such type of problems arise for instance in aerospace applications, in which it is desired to trade-off fuel consumption and state regulation performance, while limiting as much as possible the control activation time. Closed-loop stability is guaranteed by embedding in the design a terminal set and a terminal cost, both of which are periodically time-varying. Differently from quadratic MPC, the terminal cost is in the form of a weighted 2-norm. A systematic method to construct such a function is presented. The proposed design is demonstrated on a spacecraft rendezvous case study with periodic dynamics.

I. INTRODUCTION

Control systems with periodically time-varying dynamics play a prominent role in many engineering problems. While it is well known that periodic controllers can be usefully applied to linear time-invariant (LTI) plants (see e.g., [1], [2]), the motivation for resorting to periodic control usually comes from the periodicity of the system model or in order to enforce a periodic behavior. Examples can be found in robot guidance [3], vibration suppression [4], aerospace applications [5], [6], [7], and many other contexts.

Periodic systems have been extensively studied in the control literature. Most of the classical stability results for LTI systems have been extended to the linear time-periodic (LTP) case (see [8] for a comprehensive overview). The extension typically involves an increased computational burden. In order to cope with this issue, some studies have focused on receding-horizon control of unconstrained LTP systems [9], [10].

In recent years, the focus has shifted to constrained LTP problems. In [11], a Model Predictive Control (MPC) strategy is proposed for such problems. Closed-loop stability is achieved by including an ellipsoidal terminal set and quadratic terminal cost, both of which are periodically time-varying, in the design. These are computed via the solution of a linear matrix inequality (LMI) problem. A similar approach is taken in [12], where the adopted terminal set is polyhedral and the LMI condition is replaced by the solution of a periodic Riccati equation. The stability proof for these methods follows the same line as in most MPC schemes [13]. It boils down to showing that the terminal set is positively invariant and that the optimal cost decreases monotonically over time. A less conservative stability condition, which

requires the optimal cost to decrease periodically rather than monotonically, is investigated in [14].

A common trait of the aforementioned MPC strategies is that the cost function to be optimized is quadratic. This is indeed a standard setting in the MPC literature. However, despite its widespread adoption, a quadratic cost does not always represent the actual performance objective for the controlled process. In space applications, for instance, it is often more appropriate to cast the performance requirements in terms of a sum of vector norms [15]. Furthermore, such formulation usually yields optimal solutions involving a sparse control activation.

While the optimization of a nonquadratic convex cost does not raise particular challenges within the MPC framework, the closed-loop stability analysis is typically more involved. In particular, for vector-norm-based formulations, enforcing the cost decrease condition requires to find a suitably weighted vector norm, to be used as a Lyapunov function [15], [16], [17].

Motivated by the relevance of periodic models in space applications, this paper develops a sum-of-norms MPC strategy for constrained LTP systems. A sum of 2-norms cost function is considered, which allows one to trade off state regulation performance and fuel consumption in a thrust-vectoring propulsion scenario. Closed-loop stability is guaranteed by extending the results in [15] to the periodic case. In particular, a periodically weighted 2-norm of the system state is employed as a terminal cost. A systematic method to construct such weights from the solution of a periodic Lyapunov equation, is presented.

The proposed MPC scheme is validated on a case study of a spacecraft rendezvous mission (see, e.g., [18], [19]). The considered model takes into account periodic effects due to the second zonal harmonic of the geopotential and to solar eclipses. Simulations show that the controller achieves finite time convergence and control sparsity, which are two desirable properties in this application. Moreover, it is observed that incorporating eclipse constraints in the design provides a far better performance compared to neglecting this possibility.

The rest of the paper is organized as follows. The sum-of-norms MPC problem is described in Section II. Sufficient conditions for closed-loop stability are put forth in Section III. The rendezvous case study is discussed in Section IV and conclusions are drawn in Section V.

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Notation and preliminaries

For a real vector $x \in \mathbb{R}^n$, $\|x\|$ denotes its 2-norm, while for a matrix $P \in \mathbb{R}^{n \times n}$, $\|P\|$ denotes the induced matrix 2-norm. For a symmetric real matrix P , $\lambda_m(P)$ indicates its minimum eigenvalue, and $P > 0$ ($P \geq 0$) denotes (semi)positive definiteness. The set $\mathbb{N} = \{0, 1, \dots\}$ is the set of nonnegative integers. The following definitions are also in order.

Definition 1: For given $N \in \mathbb{N}$, a matrix sequence M_k , $k \in \mathbb{N}$, is termed N -periodic if it satisfies

$$M_k = M_{k+N} \quad \forall k \in \mathbb{N}. \quad (1)$$

Definition 2: Given N -periodic matrix sequences A_k and symmetric $R_k \geq 0$, a symmetric matrix sequence $P_k > 0$ is said to be a solution of the N -periodic Lyapunov equation

$$A_k^T P_{k+1} A_k - P_k + R_k = 0 \quad (2)$$

if (2) holds for all $k \in \mathbb{N}$. Clearly, if symmetric positive definite matrices $P_0, \dots, P_N$ with $P_N = P_0$ satisfy (2) for $k = 0, \dots, N-1$, then the sequence P_k such that $P_k = P_{(k \bmod N)}$ is a solution of (2) for all $k \in \mathbb{N}$.

II. PROBLEM FORMULATION

This paper deals with periodic discrete-time linear systems of the form

$$x(k+1) = A_k x(k) + B_k u(k), \quad (3)$$

where $k \in \mathbb{N}$, $x(k) \in \mathbb{R}^n$ is the system state, $u(k) \in \mathbb{R}^m$ is the control input, and system matrices $A_k \in \mathbb{R}^{n \times n}$ and $B_k \in \mathbb{R}^{n \times m}$ are N -periodic for a given period N .

The control problem addressed in this paper is to asymptotically stabilize the origin of system (3) via an MPC control scheme in which a sum-of-norms objective function and 2-norm control bounds are employed. Such a choice is particularly suited to spacecraft maneuvering applications, where system (3) describes the linearized orbital dynamics (see, e.g., [20]). In such applications, a trade-off between fuel consumption and state regulation performance must be achieved. Furthermore, actuator limitations have to be taken into account. In particular, we consider the thrust vectoring scenario, where constraints on maximum deliverable thrust can be expressed as $\|u(k)\| \leq 1$, $\forall k \in \mathbb{N}$, and fuel consumption over a given time interval $\mathbb{I} \subseteq \mathbb{N}$ is proportional to $\sum_{k \in \mathbb{I}} \|u(k)\|$ (see [15] for details).

The above considerations naturally lead to an MPC design framework based on the solution, at each time instant $k \in \mathbb{N}$, of the following trajectory optimization problem

$$\begin{aligned} \min_{\hat{\mathcal{U}}_k} & J_k(\hat{\mathcal{U}}_k) \\ \text{s.t.} & \hat{x}_k(0) = x(k) \\ & \hat{x}_k(j+1) = A_{k+j} \hat{x}_k(j) + B_{k+j} \hat{u}_k(j) \\ & \|\hat{u}_k(j)\| \leq 1 \quad j = 0 \dots H-1 \\ & \|S_{k+H} \hat{x}_k(H)\| \leq 1, \end{aligned} \quad (4)$$

where H is a given time horizon length, $\hat{x}_k(j)$ denotes the predicted state j steps ahead of k , and the decision variables are the elements of the control sequence

$$\hat{\mathcal{U}}_k = \{\hat{u}_k(0), \dots, \hat{u}_k(H-1)\}. \quad (5)$$

The objective function $J(\hat{\mathcal{U}}_k)$ is chosen as

$$J_k(\hat{\mathcal{U}}_k) = \sum_{j=0}^{H-1} \left\{ \|Q \hat{x}_k(j)\| + \|\hat{u}_k(j)\| \right\} + \|W_{k+H} \hat{x}_k(H)\|. \quad (6)$$

In (4),(6), Q is a full-rank matrix, while W_{k+H} and S_{k+H} are full-rank matrices belonging to N -periodic sequences W_k and S_k , respectively. Notice that (6) differs from the widely used quadratic cost in the fact that it employs 2-norms. In the spacecraft maneuvering context, the component $\sum_j \|\hat{u}_k(j)\|$ accounts for fuel consumption, while matrix Q establishes a trade-off between the latter requirement and tracking performance. The matrix sequence W_k defines the terminal cost in (6), while S_k characterizes the terminal constraint in (4). These are instrumental to stability assessment for the proposed MPC scheme.

Problem (4) is a convex second order cone program (SOCP) which is solved at each discrete-time step k . Then, in the standard receding horizon fashion, the first element of the optimal solution

$$\hat{\mathcal{U}}_k^* = \{\hat{u}_k^*(0), \dots, \hat{u}_k^*(H-1)\} \quad (7)$$

is applied to the system, i.e.,

$$u(k) = \hat{u}_k^*(0). \quad (8)$$

Furthermore, let $V_k(x(k))$ denote the optimal cost of problem (4), i.e.,

$$V_k(x(k)) = J_k(\hat{\mathcal{U}}_k^*). \quad (9)$$

Asymptotic stability of the closed-loop system (3) with the control law (8) can be assessed along the lines of [13] by ensuring that problem (4) is recursively feasible and that its optimal cost $V_k(x(k))$ is strictly decreasing along the closed-loop system trajectories. This can be accomplished by suitably designing the N -periodic matrix sequences S_k and W_k which define the terminal constraint and the terminal cost, respectively, as detailed in the next section.

III. STABILITY RESULT

Assume that system (3) is stabilizable via N -periodic linear feedback [8], and consider an auxiliary fictitious asymptotically stabilizing control law

$$u(k) = -K_k x(k), \quad (10)$$

where $K_k \in \mathbb{R}^{m \times n}$ is a given N -periodic matrix sequence. The stabilizing periodic feedback K_k can be computed, for instance, by solving a periodic Riccati equation or an equivalent set of linear matrix inequalities, see, e.g., [8], [11]. The resulting closed-loop system is given by

$$x(k+1) = (A_k - B_k K_k) x(k) = A_k^c x(k), \quad (11)$$

which is clearly N -periodic. The control law (10) is exploited in the sequel to design the sought matrix sequences S_k and W_k .

The following result characterizes a possible choice of S_k ensuring that problem (4) is recursively feasible.

Proposition 1: Let $Z_k^T Z_k$ be a solution of the periodic Lyapunov equation

$$(A_k^{cl})^T Z_{k+1}^T Z_{k+1} A_k^{cl} - Z_k^T Z_k + C_k = 0, \quad (12)$$

where C_k is a given symmetric positive definite N -periodic matrix sequence. In (4), let S_k be defined as

$$S_k = \left(\max_{i \in \{0, \dots, N-1\}} \|K_i Z_i^{-1}\| \right) \cdot Z_k. \quad (13)$$

Suppose problem (4) is feasible at time k_0 . Then, it is also feasible for all $k > k_0$ along the trajectories of system (3) under the MPC control law (8).

Proof: Let us define the periodically time-varying region

$$\mathcal{C}_k = \{x \in \mathbb{R}^n : \|K_k x\| \leq 1\}, \quad (14)$$

i.e., the set of states $x(k)$ for which the control law (10) satisfies the constraint $\|u(k)\| \leq 1$. From (12), and positive definiteness of C_k it follows that $\|Z_k x\| \leq 1$ implies $\|Z_{k+1} A_k^{cl} x\| \leq 1$. This property is preserved under a constant scaling, hence, defining the periodically time-varying region

$$\mathcal{X}_k = \{x \in \mathbb{R}^n : \|S_k x\| \leq 1\}, \quad (15)$$

we get that $x \in \mathcal{X}_k$ implies $A_k^{cl} x \in \mathcal{X}_{k+1}$. Thus, $x(k) \in \mathcal{X}_k$ implies $x(k+1) \in \mathcal{X}_{k+1}$ under the control law (10). Moreover, the largest sublevel set of $\|Z_k x\| \leq 1$ in which the constraint $\|K_k x\| \leq 1$ holds, is given by $(\|K_k Z_k^{-1}\|) \|Z_k x\| \leq 1$. Hence, defining S_k as in (13), we get that $\|S_k x\| \leq 1$ implies $\|K_k x\| \leq 1$, i.e., $\mathcal{X}_k \subseteq \mathcal{C}_k$ for all $k \in \mathbb{N}$. Therefore, if $x(k_0) \in \mathcal{X}_{k_0} \subseteq \mathcal{C}_{k_0}$, then the control law (10) ensures $\|u(k)\| \leq 1$ for all $k \geq k_0$.

By assumption, (4) is feasible at time k_0 . Now, suppose (4) is feasible at time k and let $\hat{x}_k^*(j)$ be the value of $\hat{x}_k(j)$ at the optimum. Then, $\hat{x}_k^*(H) \in \mathcal{X}_{k+H}$ implying $\hat{x}_k^*(H) \in \mathcal{C}_{k+H}$. From the above arguments, it follows that

$$\bar{U}_{k+1} = \{\hat{u}_k^*(1), \dots, \hat{u}_k^*(H-1), -K_{k+H} \hat{x}_k^*(H)\} \quad (16)$$

is a feasible solution of (4) at time $k+1$. By induction, the result is proved. ■

The following proposition, which is an extension to the periodic case of a result in [15] for LTI systems, characterizes a possible choice of W_k guaranteeing that the cost decrease condition is met.

Proposition 2: Let $Y_k^T Y_k$ be a solution to the periodic Lyapunov equation

$$(A_k^{cl})^T Y_{k+1}^T Y_{k+1} A_k^{cl} - Y_k^T Y_k + (\|Q\| + \|K_k\|) D_k = 0, \quad (17)$$

where D_k is a given N -periodic positive definite symmetric matrix sequence. Define

$$W_k = \left(\min_{i \in \{0, \dots, N-1\}} \frac{\lambda_m(D_i)}{\|Y_{i+1} A_i^{cl}\| + \|Y_i\|} \right)^{-1} \cdot Y_k \quad (18)$$

Then, the sequence of optimal objectives $V_k(x(k))$ of problem (4), (13), (18) satisfies

$$V_{k+1}(x(k+1)) - V_k(x(k)) \leq -\|Qx(k)\| \quad (19)$$

along the trajectories of system (3) with the control law (8).

Proof: The fact that $Y_k^T Y_k$ solves (17) implies

$$x^T A_k^{clT} Y_{k+1}^T Y_{k+1} A_k^{cl} x - x^T Y_k^T Y_k x + x^T (\|Q\| + \|K_k\|) D_k x = 0 \quad \forall x \in \mathbb{N}, \quad \forall x \in \mathbb{R}^n, \quad (20)$$

which in turn can be rewritten as

$$\|Y_{k+1} A_k^{cl} x\| - \|Y_k x\| = \frac{-x^T (\|Q\| + \|K_k\|) D_k x}{\|Y_{k+1} A_k^{cl} x\| + \|Y_k x\|}. \quad (21)$$

Using a standard upper bound for the expression on the right hand side of (21), one gets

$$\|Y_{k+1} A_k^{cl} x\| - \|Y_k x\| \leq \frac{-\lambda_m(D_k) (\|Q\| + \|K_k\|)}{\|Y_{k+1} A_k^{cl}\| + \|Y_k\|} \|x\|. \quad (22)$$

Taking into account (18), it follows from (22) that

$$\|W_{k+1} A_k^{cl} x\| - \|W_k x\| \leq -(\|Q\| + \|K_k\|) \|x\| \quad (23)$$

which, being $(\|Q\| + \|K_k\|) \|x\| \geq \|Qx\| + \|K_k x\|$, implies

$$\|W_{k+1} A_k^{cl} x\| - \|W_k x\| \leq -\|Qx\| - \|K_k x\| \quad (24)$$

for all $x \in \mathbb{R}^n$ and $k \in \mathbb{N}$.

Consider the feasible solution $\bar{U}_{k+1}$ in (16) at time $k+1$. One has that

$$\begin{aligned} J_{k+1}(\bar{U}_{k+1}) - V_k(x(k)) &= -\|Qx(k)\| - \|u_k^*(0)\| \\ &\quad + \|Q\hat{x}_k^*(H)\| + \|K_{k+H} \hat{x}_k^*(H)\| \\ &\quad + \|W_{k+H+1} A_{k+H}^{cl} \hat{x}_k^*(H)\| - \|W_{k+H} \hat{x}_k^*(H)\|. \end{aligned} \quad (25)$$

By combining (25) with (24) evaluated at time $k+H$ with $x = \hat{x}_k^*(H)$, it follows that

$$\begin{aligned} J_{k+1}(\bar{U}_{k+1}) - V_k(x(k)) &\leq -\|Qx(k)\| - \|u_k^*(0)\| \\ &\leq -\|Qx(k)\|. \end{aligned} \quad (26)$$

Finally, since $\bar{U}_{k+1}$ is in general suboptimal at time $k+1$, from (9) and (26) we have

$$V_{k+1}(x(k+1)) - V_k(x(k)) \leq -\|Qx(k)\|, \quad (27)$$

which proves the result. ■

Notice that C_k in (12) and D_k in (17) can be chosen independently. The next result states the stability property of the proposed MPC scheme.

Theorem 1: The MPC design (4)-(6) with S_k and W_k as in Propositions 1-2 renders the equilibrium point $x = 0$ of the closed-loop system (3) under (8) asymptotically stable. Furthermore, for all initial conditions $x(k_0)$ such that problem (4) is feasible, $x(k)$ converges to zero exponentially.

Proof: The result can be proven by observing that the optimal cost $V_k(x(k))$ in (9) is a Lyapunov function. Clearly, $V_k(x(k))$ is positive definite. By (6), (9), it follows that

$$c_1 \|x(k)\| \leq V_k(x(k)) \leq c_2 \|x(k)\|, \quad (28)$$

where $c_1 = \sqrt{\lambda_m(Q^T Q)}$, and c_2 can be obtained according to Proposition 2.38 in [21]. Moreover, (27) implies

$$V_{k+1}(x(k+1)) - V_k(x(k)) \leq -c_1 \|x(k)\|, \quad (29)$$

and, from (28)-(29),

$$V_{k+1}(x(k+1)) \leq c_3 V_k(x(k)),$$

with $c_3 = 1 - c_1/c_2$. Note that $0 < c_3 < 1$. Therefore,

$$\begin{aligned} \|x(k)\| &\leq \frac{1}{c_1} V_k(x(k)) \leq \frac{1}{c_1} c_3^{k-k_0} V_{k_0}(x(k_0)) \\ &\leq \frac{c_2}{c_1} c_3^{k-k_0} \|x(k_0)\|, \end{aligned}$$

which, together with recursive feasibility established in Proposition 1, proves the result. ■

IV. APPLICATION TO SPACECRAFT RENDEZVOUS

In this section, the proposed MPC design is demonstrated on a space rendezvous problem in Earth orbit, in which a controlled spacecraft equipped with a propulsion system operating in thrust vectoring mode is required to intercept an uncontrolled target. Earth's oblateness and solar eclipse effects are also considered.

A. System model

The adopted model of the orbital motion, which is taken from [22], is briefly summarized hereafter. We refer to the controlled and the target spacecraft by satellite 2 and satellite 1, respectively. Let the *relative line of nodes* be defined as the intersection of the orbital planes of the two satellites. The angle between these planes is denoted by γ . Moreover, let λ_j and θ_j be the angles formed respectively by the periaapsis of orbit j and the position vector of satellite j with respect to the relative line of nodes. The relative motion between the two satellites is parameterized by the state vector $x = [x_1, \dots, x_6]^T$, given by

$$\begin{aligned} x_1 &= \theta_2 - \theta_1 \\ x_2 &= (n_2 - n_1)/n_1 \\ x_3 &= e_2 \cos(\theta_2 - \lambda_2) - e_1 \cos(\theta_2 - \lambda_1) \\ x_4 &= e_2 \sin(\theta_2 - \lambda_2) - e_1 \sin(\theta_2 - \lambda_1) \\ x_5 &= \tan(\gamma/2) \cos \theta_2 \\ x_6 &= \tan(\gamma/2) \sin \theta_2, \end{aligned} \quad (30)$$

where n_j , e_j are the mean motion and the eccentricity of satellite j , respectively. In terms of the parameters (30), the rendezvous condition is $x = 0$.

We restrict our attention to short-range rendezvous maneuvers and satellites in near-circular orbits ($e_1 \approx 0$). The Earth is modeled as an oblate spheroid. Within this setting, the linearized continuous-time dynamics of the state vector (30) are given by

$$\dot{x} = A(t)x + Bu, \quad (31)$$

where u , is the control acceleration delivered by satellite 2, expressed in the local-vertical-local-horizontal frame. The state matrix $A(t)$ in (31) takes on the form

$$A(t) = A_{Kep} + A_{J_2}(t),$$

where A_{Kep} models the contribution of spherical gravity, and $A_{J_2}(t)$ describes the secular effects of the second zonal harmonic J_2 of the geopotential. The matrix A_{Kep} reads

$$A_{Kep} = n_1 \begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The matrix $A_{J_2}(t)$ turns out to be

$$A_{J_2}(t) = \kappa \begin{bmatrix} 0 & 14c/3 & 2c & 0 & a_{15}(t) & a_{16}(t) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 \\ 0 & a_{52}(t) & 0 & 0 & a_{55}(t) & a_{56}(t) \\ 0 & a_{62}(t) & 0 & 0 & a_{65}(t) & a_{66}(t) \end{bmatrix},$$

where $c = 3 \cos^2 i_1 - 1$, i_1 is the inclination of satellite 1, $\kappa = 3 J_2 R_e^2 n_1^{7/3} / (4 \mu^{2/3})$ is a constant proportional to J_2 , R_e is the Earth radius, μ is the gravitational parameter, and

$$\begin{aligned} a_{15}(t) &= -14 \sin(2i_1) \cos \varphi(t) \\ a_{16}(t) &= -14 \sin(2i_1) \sin \varphi(t) \\ a_{52}(t) &= -7 \sin(2i_1) \sin \varphi(t)/6 \\ a_{55}(t) &= \sin^2 i_1 \sin(2\varphi(t)) \\ a_{56}(t) &= 2 \sin^2 i_1 [\sin^2 \varphi(t) + 4] - 6 \\ a_{62}(t) &= 7 \sin(2i_1) \cos \varphi(t)/6 \\ a_{65}(t) &= 2 \sin^2 i_1 [\sin^2 \varphi(t) - 5] + 6 \\ a_{66}(t) &= -\sin^2 i_1 \sin(2\varphi(t)). \end{aligned}$$

The angle $\varphi(t)$ is the argument of latitude of satellite 1. It is T -periodic in t , where $T = 2\pi/[n_1 + 2\kappa(4 \cos^2 i_1 - 1)]$ is the satellite 1 nodal period. The input B matrix in (31) is given by

$$B = \frac{1}{(\mu n_1)^{1/3}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}.$$

For the purpose of digital control design, system (31) is discretized with a sampling interval t_s (chosen as a suitable submultiple of T), yielding a periodic discrete-time model of the form (3). Moreover, the dimensional unit of the state vector $x(k)$ of system (3) is normalized by the constant (multiplicative) factor m_2/F_{max} , where F_{max} is the maximum deliverable thrust and m_2 is the mass of satellite 2. In this setting, the input vector $u(k)$ is related to the actual thrust $F(k)$ by the identity $u(k) = F(k)/F_{max}$. Hence, thrust constraints are expressed as $\|u(k)\| \leq 1$. Then, the rendezvous control problem is formulated as in Section II.

Besides capturing J_2 , the considered LTP framework can be used to account for power limitations affecting the spacecraft. In particular, for low-thrust propelled spacecraft,

the power available on board is typically not sufficient to operate the engine whenever the spacecraft is shadowed from the Sun, i.e., during solar eclipses [23]. In this regard, it should be noticed that solar eclipses occur almost periodically. Indeed, within the rendezvous maneuver time scale (usually less than few days), the occurrence of eclipses has approximately the same period of system (3). Then, the effect of engine cut-offs the system dynamics can be adequately modeled by enforcing

$$B_k = 0 \quad \forall k : k \pmod{N} \in \mathcal{E}, \quad (32)$$

where $\mathcal{E}$ is the set of time indexes in which satellite 2 is shadowed from the Sun, within the discrete-time period $N = T/t_s$. Following this approach, one obtains a periodic model of the form (3), in which some of the input matrices $B_k = B_{k+N}$ are zero. Hereafter, simulation results are reported for such description.

B. Simulation results

The considered rendezvous scenario is as follows. The target spacecraft lies on a circular orbit with a semi-major axis equal to 7387.1 km and an inclination of 81 deg. At the beginning of the rendezvous process, the controlled spacecraft is located 1 km below the target, out-of-phase by -0.155 deg. The initial inclination difference between the two spacecraft amounts to -0.001 deg. These parameters correspond to an initial inter-satellite separation of approximately 20 km. The controlled spacecraft has the following specifications: maximum deliverable thrust $F_{max} = 15$ mN and wet mass $m_2 = 100$ kg.

The periodic system (3),(32) turns out to be controllable according the results in [8]. The rendezvous control problem is tackled by using the MPC scheme (4). The gain matrix K_k in (10) is set as the periodic LQR state feedback matrix

$$K_k = (B_k^T \Psi_{k+1} B_k + I)^{-1} B_k^T \Psi_{k+1} A_k, \quad (33)$$

where Ψ_k is the solution to the periodic discrete-time ARE

$$A_k^T \Psi_{k+1} A_k - A_k^T \Psi_k B_k (I + B_k^T \Psi_k B_k)^{-1} B_k^T \Psi_k A_k - \Psi_k + Q^T Q = 0.$$

According to (32)-(33), the auxiliary control policy (10) is such that $u(k) = 0$ during eclipses. The matrix S_{k+H} in (4) is chosen as in Proposition 1 with $C_k = Q^T Q + K_k^T K_k$. The terminal weight W_{k+H} in (4) is set as in Proposition 2, where for simplicity the matrix D_k is taken as the identity matrix for all k . A systematic method for exploiting the degrees of freedom provided by D_k for performance optimization is currently under investigation.

The sampling interval for the discretization of model (31) is taken as $t_s = T/32$ (corresponding to about 3 minutes), while the prediction horizon of the controller is set to $H = 48$. Such a long horizon is adopted to effectively deal with the limited control authority provided by the actuation system, which in this study consists of a low-thrust engine. The MPC problem is solved by using the package CVX and the commercial solver Gurobi. On a standard laptop, the solver time amounts on average to 0.7 s, i.e., a negligible

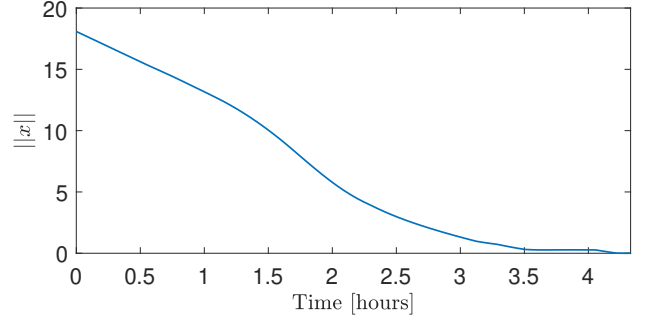


Fig. 1. Magnitude of the system state trajectories.

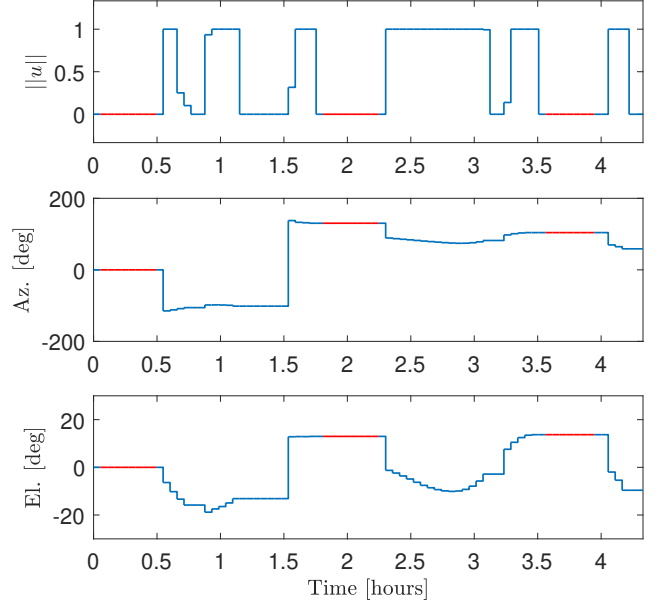


Fig. 2. Magnitude, azimuth and elevation of the control input provided by the MPC scheme.

fraction of the sampling time. The rendezvous maneuver is simulated on a nonlinear truth model accounting for Earth asphericity, atmospheric drag, luni-solar gravity, and solar radiation pressure perturbations. Figure 1 shows the evolution of the 2-norm of the relative states resulting from the simulation. It can be seen that the controller achieves finite time convergence. This is in line with what has been observed in [15] for the LTI case. The spherical coordinate components (magnitude, azimuth, elevation) of the control input $u(k)$ are reported in Figure 2 (the azimuth and elevation signals are held constant whenever $u(k) = 0$). It can be seen that the input constraint $\|u(k)\| \leq 1$ is always satisfied. Moreover, the command is idle during eclipse phases (highlighted in red). The control policy displays a sparse engine activation pattern, which is a desirable feature in order to limit the engine wear.

The above results have been compared to those obtained by neglecting (32) in the MPC problem and enforcing engine cut-offs (i.e., $u(k) = 0$ within eclipse phases) a posteriori. Such heuristic is representative of a scenario in which the eclipse location is not known/estimated in advance. Figure 3 depicts the resulting spacecraft trajectories, expressed in the

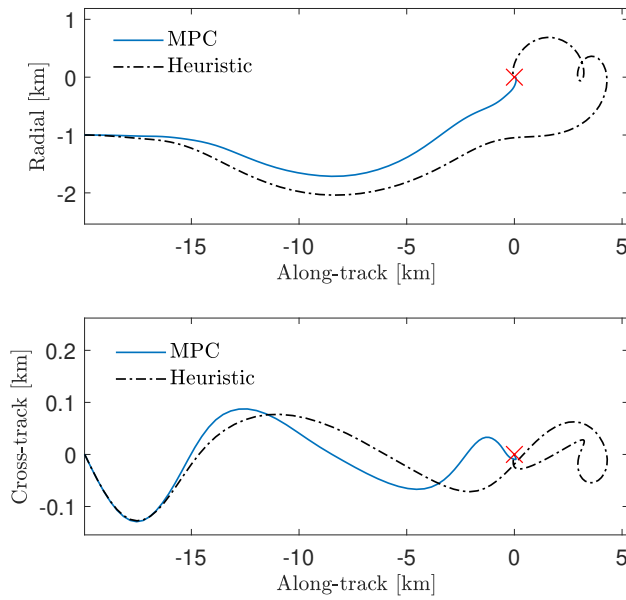


Fig. 3. Simulated trajectories in the local-vertical-local-horizontal frame: radial versus along-track displacement (top) and cross-track vs along-track displacement (bottom).

local-vertical-local-horizontal frame centered at the target. Observe that the trajectory generated by the heuristic still converges to the set-point, which indicates some degree of robustness of the controller against plant-model mismatch. Nevertheless, it is apparent that the inclusion of eclipse effects in the MPC design greatly improves the regulation performance.

V. CONCLUSIONS

An MPC strategy has been derived for linear time-periodic systems with norm-bounded inputs. The proposed design allows one to trade off fuel consumption and tracking performance through the minimization of a sum-of-norms cost function. Closed-loop exponential stability is established by adopting a suitable definition of the terminal set and of the terminal penalty term. The control scheme has been applied to a spacecraft rendezvous problem, exploiting the flexibility provided by the LTP framework. Simulation results show that the rendezvous objective is achieved satisfactorily, in spite of engine cut-offs due to eclipsing. Future research will focus on the optimization of the tuning parameters of the controller and on the development of results for more complex scenarios accounting for additional control specifications such as trajectory constraints.

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