On the classif cation of certain geproci sets

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Abstract

In this short note we develop new methods toward the ultimate goal of classifying geproci sets in $\mathbb{P}^3$. We apply these methods to show that among sets of 16 points distributed evenly on 4 skew lines, up to projective equivalence there are only two distinct geproci sets. We give dif erent geometric distinctions between these sets. The methods we develop here can be applied in a more general set-up; this is the context of follow-up work [?].

1 Introduction

The study of geproci sets was initialized in one of the previous workshops on Lefschetz properties held in Levico Terme (Italy) in 2018. In the present note we work exclusively over the f eld $\mathbb{C}$ of complex numbers.

Def nition 1.1 (A geproci set of points). We say that a set of points $Z \subset \mathbb{P}^N_{\mathbb{C}}$ with $N \geq 3$ is **geproci** (for GEneral PROjection is a Complete Intersection), if its general projection to a hyperplane is a complete intersection.

We say that a geproci set is **trivial**, if it is already contained in a hyperplane. From now on we consider only nontrivial geproci sets.

So far nontrivial geproci sets have been discovered only in $\mathbb{P}^3$. They project to a plane, where their images are the intersection points of two curves of degrees a and b (equivalently: the ideal of the projection has exactly two generators: one of degree a and another of degree b). We assume that $a \leq b$ and we refer to such sets of points as (a, b)-geproci.

Example 1.2 (A grid). Assume that we have two positive integers $a \leq b$. Let $Z \subset \mathbb{P}^3$ be a grid, i.e., the set of all intersection points among lines in two sets $\mathsf{L} = \{L_1, \ldots, L_a\}$ and $\mathsf{M} = \{M_1, \ldots, M_b\}$ such that lines from the same set, either L or M, are pairwise skew but any two lines from distinct sets intersect in a point. It is elementary to see that a grid is an (a, b)-geproci set for all values of a, b and the set is nontrivial for $a, b \geq 2$ and it is contained in a unique quadric for $a, b \geq 3$.

Grids exist for any values of a and b. They were studied extensively in [?]. Here we are interested in geproci sets which are not grids but half grids.

Def nition 1.3 (A half grid). We say that a nontrivial (a, b)-geproci set $Z \subset \mathbb{P}^3$ is a **half grid**, if it is not a grid but one of the curves determining its general projection as a complete intersection can be taken as a union of lines.

Keywords: complete intersection, geproci sets, half grids, projections.
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