$\mathbb{P}^3_{\mathbb{C}}$

$\underset{4\times 4}{M} = \begin{pmatrix} \end{pmatrix}$

rango $M = \begin{cases} 4 & \\ 3 & \text{coni} \\ 2 & \text{2 piani} \\ 1 & \text{1 piano doppio} \end{cases}$

Rango 3 $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad x_1^2 + x_2^2 + x_3^2 = 0$ $\quad$ superficie quadrica S

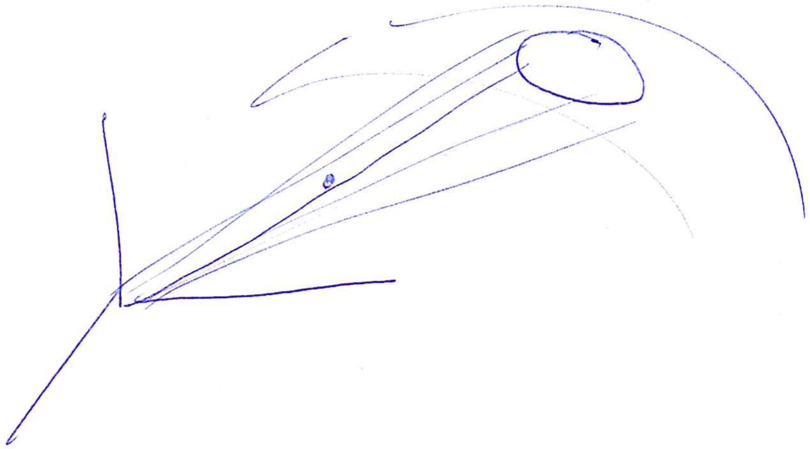
$x^2 + y^2 + z^2 = 0$

$P = (a, b, c) \in S$
$\Downarrow$
$Q = (\alpha a, \alpha b, \alpha c) \in S$

$\alpha^2 a^2 + \alpha^2 b^2 + \alpha^2 c^2 =$

$\alpha^2 (\underbrace{a^2 + b^2 + c^2}_{\Uparrow\ P \in S}) = \alpha^2 0 = 0$

$\Rightarrow$ Cono di vertice l'origine



Rango 2

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right)$$

$x^2+y^2=0$

coppia di piani

$x+iy=0$

$x-iy=0$

Rango 4

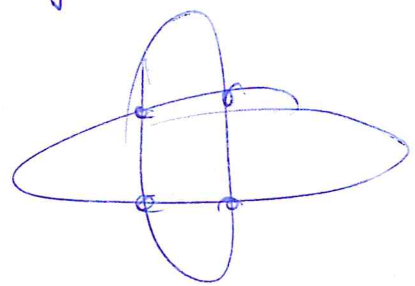
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

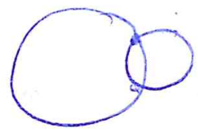
$x^2+y^2+z^2-1=0$

sfera di centro l'origine e raggio 1.

Intersezione di 2 coniche

$\begin{cases} 2° \text{ grado} \\ 2° \text{ grado} \end{cases} \Rightarrow$ grado $= 4 \Rightarrow$ 4 punti





Intersezione di 3 superfici quadriche

$\begin{cases} 2° \text{ grado} \\ 2° \text{ grado} \\ 2° \text{ grado} \end{cases} \Rightarrow$ grado 8

$\mathbb{P}^3_{\mathbb{C}}$ $\quad x_0x_3 - x_1x_2 = 0$ $\quad z - xy = 0$ $\quad \boxed{z = xy}$

$$\det\begin{pmatrix} x_0 & x_1 \\ x_2 & x_3 \end{pmatrix}$$

$\mathbb{A}^3_{\mathbb{R}}$

iperboloide

$$\begin{pmatrix} 0 & -1/2 & 0 & 0 \\ -1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1/2 & 0 \end{pmatrix}$$

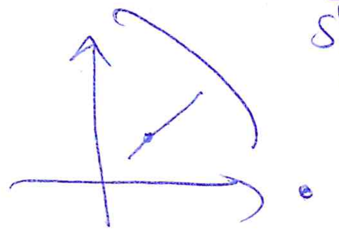
rango 4 in $\mathbb{P}^3_{\mathbb{C}}$ $\iff$ $\mathbb{P}^1_{\mathbb{C}} \times \mathbb{P}^1_{\mathbb{C}}$

$\mathbb{P}^2_{\mathbb{C}}$ $\quad (x, y)$ $\quad \left(\frac{x_1}{x_0}, \frac{x_2}{x_0}\right)$

stesso denom.

(x, y) $\quad \left(\frac{x'}{x''}, \frac{y'}{y''}\right)$

$((x', x'')\,(y', y''))$



$(1, 0, 0)$

$(2, 0, 0)$

$(x, 0, 0)$

$$(x', x'', y', y'') \sim (\bar{x}', \bar{x}'', \bar{y}', \bar{y}'')$$

$$\frac{x'}{x''} = \frac{\bar{x}'}{\bar{x}''} \qquad \frac{y'}{y''} = \frac{\bar{y}'}{\bar{y}''}$$

$$\exists\, \lambda_1\ \lambda_2 \qquad \underline{(x', x'') = \lambda_1(\bar{x}', \bar{x}'') \qquad (y', y'') = \lambda_2(\bar{y}', \bar{y}'')}$$

$$(x', x'', y', y'') \rightsquigarrow (x'y', x'y'', x''y', x''y'')$$

$$(\bar{x}', \bar{x}'', \bar{y}', \bar{y}'') \rightsquigarrow (\bar{x}'\bar{y}', \bar{x}'\bar{y}'', \bar{x}''\bar{y}', \bar{x}''\bar{y}'') \Rightarrow$$

$$x_0 \quad x_1 \quad x_2 \quad x_3$$

Data la matrice simmetrica

$$M = \begin{pmatrix} 1 & k & 1 \\ k & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

e la forma bilineare associata b

1) Determinare $\forall k$ l'ortogonale di $2e_1 - 2e_2 - e_3$ risp. a b
2) Discutere la definizione o quasi-definizione di b
3) Studiare la conica associata a M in $\mathbb{P}^2_{\mathbb{C}}$ e $\mathbb{A}^2_{\mathbb{R}}$

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① $$\begin{pmatrix} 2 & -2 & -1 \end{pmatrix} \begin{pmatrix} 1 & k & 1 \\ k & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \qquad (2-2k-1,\ 2k-1,\ 0) \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$(1-2k)a + (2k-1)b = 0$$

$$(2k-1)(b-a) = 0$$

$k \neq \frac{1}{2}$ $\quad a=b \quad$ $v^\perp = \{(a,a,c) : a,c \in \mathbb{R}\} = \mathcal{L}((1,1,0),(0,0,1))$

$k = 1/2 \qquad v^{\perp} = \mathbb{R}^3$

$v \in Rad(b)$

(2) $\begin{pmatrix} 1-\alpha & k & 1 \\ k & -\alpha & 1 \\ 1 & 1 & -\alpha \end{pmatrix}$

$(1-\alpha)(\alpha^2-1) - k(-k\alpha-1) + (k+\alpha)$

$\alpha^2 - 1 - \alpha^3 + \alpha + k^2\alpha + k + k + \alpha$

$-\alpha^3 + \underbrace{\alpha^2}_{+} + \underbrace{\alpha(k^2+2)}_{-} + 2k - 1$

(3) $\det M = 2k - 1 = 0 \iff k = 1/2$

$k \neq 1/2$ conica non deg in $\mathbb{P}^2_{\mathbb{C}}$

$k = 1/2$ 2 rette distinte in $\mathbb{P}^2_{\mathbb{C}}$

$\mathbb{A}^2_{\mathbb{R}}$

$M_{33} \quad \begin{pmatrix} 1 & k \\ k & 0 \end{pmatrix} \quad \det M_{33} = -k^2$

$k = 1/2$ 2 rette reali non //

$k = 0$ parabola

∞ punti reali

$k \neq 0, 1/2$ iperbole