$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$M = \begin{pmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{pmatrix}$$

$\mathbb{P}^2_{\mathbb{C}}$ $\mathbb{P}^2_{\mathbb{R}}$ $\mathbb{A}^2_{\mathbb{C}}$

$\mathbb{A}^2_{\mathbb{R}}$

- rango $M = 1$
  - retta all'∞ contata 2 volte $F x_0^2 = 0$ $\underline{F=0}$
  - retta affine reale contata 2 volte $(ax+by+c)^2 = 0$
- rango $M = 2$
  - due rette complesse coniugate
    - // $\det M_{33} = 0$
    - incidenti
  - due rette reali
    - retta all'∞ + un'altra retta
    - 2 rette affini // $\det M_{33} = 0$
    - 2 rette affini incidenti

$x + y + i = 0$

$x + y - i = 0$

$$(x+y+i)(x+y-i) = x^2 + 2xy + y^2 + 1 = 0$$

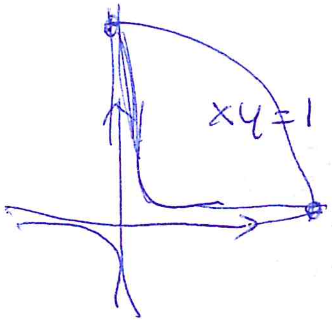
$$\overset{\det = 0}{\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \\ \cline{1-2} 0 & 0 & 1 \end{array}\right)}$$

$ax + by + c = 0$

$a'x + b'y + c' = 0$ // $\Leftrightarrow \det\begin{pmatrix} a & b \\ a' & b' \end{pmatrix} = 0$

rango $M = 3$

- parabola $\det M_{33} = 0$ ∞ punti reali
- ellisse $\det M_{33} > 0$ (tipo ellittico)
  - reale M ha autoval. concordi
  - non reale M ha autoval. discordi
- iperbole $\det M_{33} < 0$ (tipo iperbolico) ∞ punti reali



$$\begin{cases} x_1 x_2 = x_0^2 \\ x_0 = 0 \end{cases}$$

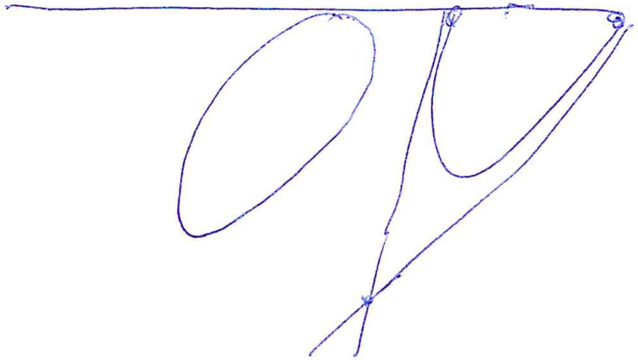
$x_1 x_2 = 0$ $(1, 0, 0)$

$(0, 1, 0)$

$$x^2 + y^2 - 2x + y - 4 = 0$$

$$x_1^2 + x_2^2 - 2x_1x_0 - x_2x_0 - 4x_0^2 = 0$$

$$x_1^2 + x_2^2 = 0 \qquad x_1 = \pm i x_2$$



$(i, 1, 0)$
$(-i, 1, 0)$ punti ciclici

Traslazioni

$$\begin{cases} X = x - a \\ Y = y - b \end{cases}$$

il punto (a,b) diventa l'origine

$$\begin{cases} X_1 = x_1 - a x_0 \\ X_2 = x_2 - b x_0 \\ X_0 = x_0 \end{cases}$$

$$A x_1^2 + B x_1 x_2 + C x_2^2 + D x_0 x_1 + E x_0 x_2 + F x_0^2 = 0$$

$$\begin{cases} x_1 = X_1 + a X_0 \\ x_2 = X_2 + b X_0 \\ x_0 = X_0 \end{cases}$$

$$A(X_1 + aX_0)^2 + B(X_1 + aX_0)(X_2 + bX_0) + C(X_2 + bX_0)^2 +$$
$$+ D X_0 (X_1 + aX_0) + E X_0 (X_2 + bX_0) + F X_0^2 = 0$$

$$A X_1^2 + 2aA X_1 X_0 + a^2 A X_0^2 + B X_1 X_2 + aB X_0 X_2 + bB X_1 X_0$$
$$+ Bab X_0^2 + C X_2^2 + 2bC X_0 X_2 + b^2 B X_0^2$$
$$+ D X_0 X_1 + aD X_0^2 + E X_0 X_2 + bE X_0^2 + F X_0^2$$

$$A X_1^2 + B X_1 X_2 + C X_2^2 + (2aA + bB + D) X_0 X_1 + (aB + 2bC + E) X_0 X_2$$
$$+ (a^2 A + Bab + b^2 B + aD + bE + F) X_0^2$$

rango $M = 3$ $\quad$ $\det M_{33} < 0$ $\quad$ $M_{33} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad$ $M = \begin{pmatrix} 1 & 0 & D/2 \\ 0 & -1 & E/2 \\ D/2 & E/2 & F \end{pmatrix}$

Def Data $M = \begin{pmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{pmatrix}$ si chiama CENTRO il punto le cui coord. sono soluz. di

$$\left(\begin{array}{ccc|c} A & B/2 & D/2 & 0 \\ B/2 & C & E/2 & 0 \end{array}\right)$$

Ex $x^2+y^2+ax+by+c=0$

$$\left(\begin{array}{ccc|c} 1 & 0 & a/2 & 0 \\ 0 & 1 & b/2 & 0 \end{array}\right)$$

$x_1 = -a/2\, x_0$

$x_2 = -b/2\, x_0$

$$\left(-\tfrac{a}{2} x_0 ; -\tfrac{b}{2} x_0 , x_0\right)$$

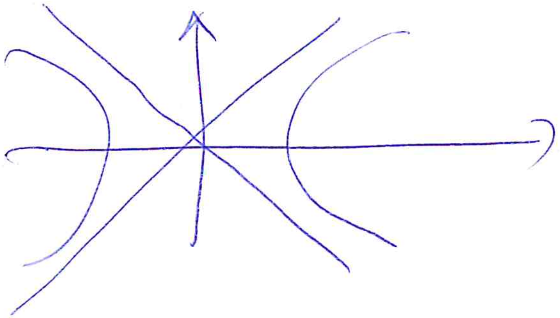
$$x_0\left(-\tfrac{a}{2}, -\tfrac{b}{2}, 1\right)$$

denom. $\leftarrow$

Portando il centro nell'origine

$$\bar{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \bar{F} \end{pmatrix}$$

$$\boxed{x^2 - y^2 + \bar{F} = 0 \qquad \text{iperbole}} \longrightarrow \infty \text{ punti reali}$$



$$y^2 = x^2 + \bar{F}$$

$$x_0(ax_1+bx_2+cx_0) = ax_0x_1+bx_0x_2+cx_0^2=0 \qquad \begin{pmatrix} 0 & 0 & a/2 \\ 0 & 0 & b/2 \\ a/2 & b/2 & c \end{pmatrix}$$

$$a\frac{x_1}{x_0}+b\frac{x_2}{x_0}+c=0$$

$$ax+by+c=0$$

Parabola $y=ax^2+bx+c$ ∞ punti reali

Ellisse $\det M \neq 0$ $\det M_{33} > 0$

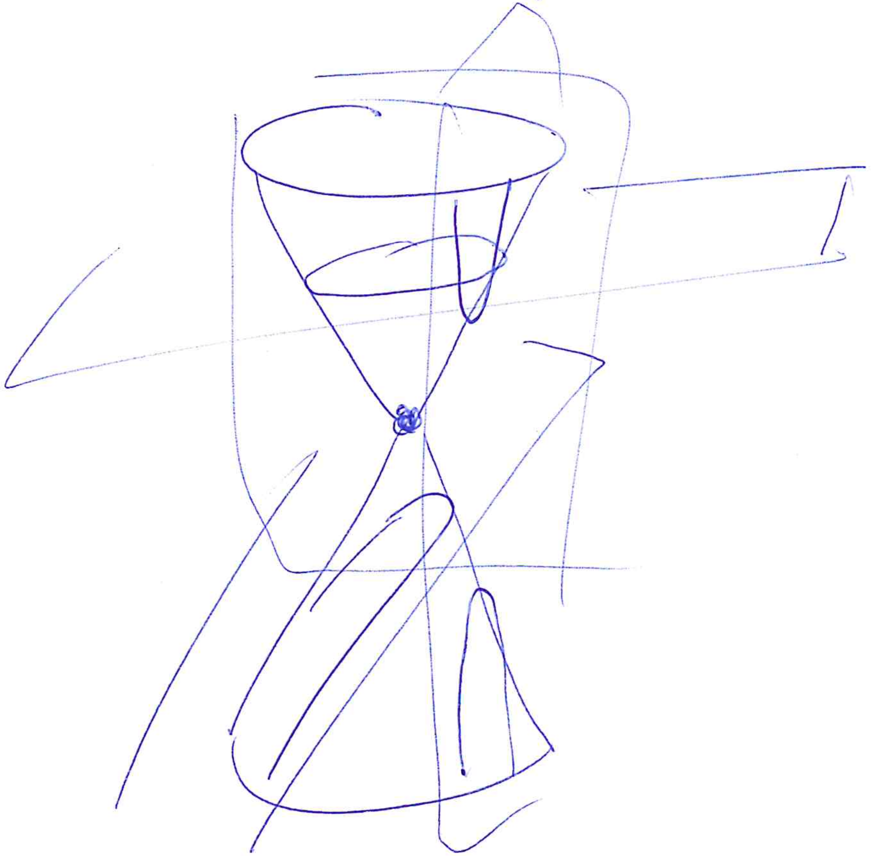
diagonalizzando M_{33} $\rightarrow$ $\begin{pmatrix} 1 & 0 & D/2 \\ 0 & 1 & E/2 \\ D/2 & E/2 & F \end{pmatrix}$

portando centro nell'origine $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \overline{F} \end{pmatrix}$ $x^2+y^2+\overline{F}=0$

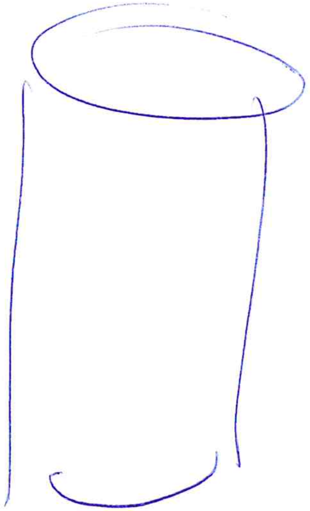
$$\text{Centro}\ \left(\det\begin{pmatrix} B/2 & D/2 \\ C & E/2 \end{pmatrix}, -\det\begin{pmatrix} A & D/2 \\ B/2 & E/2 \end{pmatrix}, \underset{\neq 0}{\det\begin{pmatrix} A & B/2 \\ B/2 & C \end{pmatrix}} \right)$$

Oss Il nome "CONICA" viene perché si tratta comunque di sezioni piane di un cono

Cono



Cono con vertice all'∞



Ex Chi è $M \begin{pmatrix} 1 & 2 & 1 \\ 2 & k & k \\ 1 & k & 0 \end{pmatrix}$ su $\mathbb{A}^2_{\mathbb{R}}$

$$x^2 + 4xy + ky^2 + 2x + 2ky = 0$$

$$\det M = \det\begin{pmatrix} 2 & 1 \\ k & k \end{pmatrix} - k \det\begin{pmatrix} 1 & 1 \\ 2 & k \end{pmatrix} = k - k(k-2)$$

$$k(1-k+2) \qquad k(3-k)$$

conica degenere $k=0$

$k=3$

conica non degenere $k \neq 0,3$

$k=0$

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 0 & 0 \\ \hline 1 & 0 & 0 \end{array}\right)$$

rango $M = 2$

det $M_{33} = -4 \neq 0$

rette non //

det $M_{33} < 0$

all'∞ ci sono

2 rette reali $\leftarrow$ 2 punti reali

$$x_1^2 + 4x_1x_2 + 2x_1x_0 = 0$$

$$x_1(x_1+4x_2+2x_0)=0 \begin{cases} x_1 = 0 & \text{asse } y \\ x_1+4x_2+2x_0=0 & \text{retta} \end{cases}$$

$k=3$ per esercizio

$k \neq 0, 3$ non degenere

$M_{33} = \begin{pmatrix} 1 & 2 \\ 2 & k \end{pmatrix}$ $\qquad$ $\det M_{33} = k-4$

$k=4$ parabola

$k<4$ iperbole

} ∞ punti reali

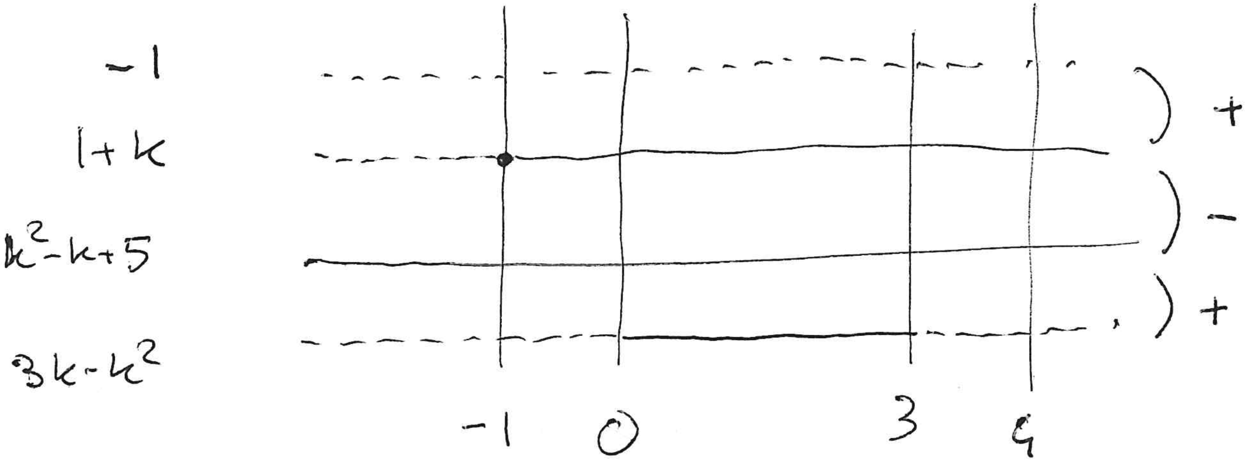
$k>4$ ellisse

$$\det \begin{pmatrix} 1-\alpha & 2 & 1 \\ 2 & k-\alpha & k \\ 1 & k & -\alpha \end{pmatrix} = (1-\alpha) \det \begin{pmatrix} k-\alpha & k \\ k & -\alpha \end{pmatrix} - 2 \det \begin{pmatrix} 2 & k \\ 1 & -\alpha \end{pmatrix} + \det \begin{pmatrix} 2 & k-\alpha \\ 1 & k \end{pmatrix}$$

$$= (1-\alpha)\left[-\alpha k + \alpha^2 - k^2\right] - 2\left[-2\alpha - k\right] + \left[2k - k + \alpha\right]$$

$$= \underline{-\alpha k} + \underline{\alpha^2} - k^2 + \underline{\alpha^2 k} \underline{- \alpha^3} + \underline{\alpha k^2} + \underline{4\alpha} + 2k + k \underline{+ \alpha}$$

$$= -\alpha^3 + \alpha^2(1+k) + \alpha(-k + k^2 + 5) + 3k - k^2$$



k^2-k+5 $\qquad k = \frac{1 \pm \sqrt{1-20}}{2}$

$3k-k^2 = k(3-k)$ $\qquad k=0 \qquad k=3$

per $k>4$ autovalori discordi ⇒ forma bilineare non definita ⇒ $\boxed{\text{ellisse reale}}$

$k<0 \qquad 0<k<3$ iperbole $\quad 3<k<4$

$k=0$ 2 rette reali

$k=3$ " "

$k=4$ parabola

$k>4$ ellisse reale

$k=3$ $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$ $x^2+4xy+3y^2+2x+6y$

$x^2+4xy+3y^2$ $x = -2y \pm \sqrt{4y^2-3y^2} = -2y \pm \sqrt{y^2} = -2y \pm y$

$x = \begin{array}{l} \nearrow -y \\ \nwarrow -3y \end{array}$

$x=-y$

$x=-3y$

$$\left(x+y\underset{4}{+}c\right)\left(x+3y+c'\right)=0$$

$$x^2+3xy+xy+3y^2+\underbrace{cx}+\overbrace{3cy}+\underbrace{c'x}+\overbrace{c'y}+cc'$$

$$x^2+4xy+3y^2+(c+c')x+(3c+c')y+cc'$$

$\left\{\begin{array}{l} c+c'=2 \\ 3c+c'=6 \\ cc'=0 \end{array}\right.$ $\left.\begin{array}{l} c+c'=2 \\ 3c+c'=6 \end{array}\right\}$ $\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 3 & 1 & 6 \end{array}\right)$ $R_2 \to R_2 - R_1$

$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & \boxed{0} & 4 \end{array}\right)$ $c'=0$

$c=2$

$$(x+y+2)(x+3y)=0$$

(x, y, z) $\left(\overset{\substack{x\\ \shortparallel}}{\frac{x_1}{x_0}}, \overset{\substack{y\\ \shortparallel}}{\frac{x_2}{x_0}}, \overset{\substack{z\\ \shortparallel}}{\frac{x_3}{x_0}} \right) \rightsquigarrow (x_1, x_2, x_3, x_0)$ $\mathbb{P}^3_{\mathbb{C}}$

punti all'∞ $\underset{\shortparallel}{\underline{x_0 = 0}}$ piano all'∞

$$0x_1 + 0x_2 + 0x_3 + 1x_0 = 0$$

$$\begin{cases} x + y = 0 \\ 2x + y = 0 \end{cases}$$

$$\begin{cases} x + y + 0z = 0 \\ 2x + y + 0z = 0 \end{cases}$$

$$2x - y + 3z + 7 = 0$$

$$2\frac{x_1}{x_0} - \frac{x_2}{x_0} + 3\frac{x_3}{x_0} + 7 = 0$$

$$2x_1 - x_2 + 3x_3 + 7x_0 = 0$$

W_1 W_2

dim 3

$$\underset{\substack{3 \quad\quad 2 \\ W_1 = W_2 \quad \text{Retta}}}{\dim(W_1 \cap W_2)} = \underset{3}{\dim(W_1)} + \underset{3}{\dim(W_2)} - \underset{\substack{3 \quad 4 \\ W_1 = W_2}}{\dim(W_1 + W_2)}$$

$$\begin{cases} 2x - y + 3z + 7 = 0 \\ 2x - y + 3z + 5 = 0 \end{cases}$$

$$\begin{cases} 2x_1 - x_2 + 3x_3 + 7x_0 = 0 \\ 2x_1 - x_2 + 3x_3 + 5x_0 = 0 \end{cases}$$

$$\left(\begin{array}{cccc|c} 2 & -1 & 3 & 7 & 0 \\ 2 & -1 & 3 & 5 & 0 \end{array}\right)$$

$R_2 \to R_1 - R_2$

$$\begin{cases} 2x_1 - x_2 + 3x_3 + 7x_0 = 0 \\ \boxed{2x_0 = 0} \text{ piano all'}\infty \end{cases}$$

$$\left(\begin{array}{cccc|c} 2 & -1 & 3 & 7 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{array}\right)$$

$$Ax^2 + Bxy + Cy^2 + Dxz + Eyz + Fz^2 + Gx + Hy + Iz + L = 0$$

$$Ax_1^2 + Bx_1x_2 + Cx_2^2 + Dx_1x_3 + Ex_2x_3 + Fx_3^2 + Gx_0x_1 + Hx_0x_2 + Ix_0x_3 + Lx_0^2 = 0$$

$$\begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_0 \\ \hline x_1 & A & B/2 & D/2 & G/2 \\ x_2 & B/2 & C & E/2 & H/2 \\ x_3 & D/2 & E/2 & F & I/2 \\ x_0 & G/2 & H/2 & I/2 & L \end{array}$$

$$x_1x_0 - x_2x_3 = 0$$

$$\parallel$$

$$\det\begin{pmatrix} x_1 & x_2 \\ x_3 & x_0 \end{pmatrix}$$