Tutti gli autoval. sono >0 $\iff$ b è def. positiva

" " <0 $\iff$ " " negativa

" " " " ≥ 0 $\iff$ b è semi-def. positiva

" " " " ≤ 0 $\iff$ " " negativa

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v autovett. di α $\qquad b(v,w) = \alpha\,(v_{\mathcal{B}} \cdot w_{\mathcal{B}})$

risp. $M_b^{\mathcal{B}}$

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se v è autoval. di α

$\alpha > 0 \Rightarrow b(v,v) > 0$

$\alpha < 0 \Rightarrow b(v,v) < 0$

$\alpha = 0 \Rightarrow v \in Rad(b)$

Es

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \qquad e_1 \quad e_2 \quad e_3$$

$$\det \begin{pmatrix} 2-\alpha & 1 & 0 \\ 1 & 1-\alpha & 1 \\ 0 & 1 & 2-\alpha \end{pmatrix} = (2-\alpha) \det \begin{pmatrix} 1-\alpha & 1 \\ 1 & 2-\alpha \end{pmatrix} - \det \begin{pmatrix} 1 & 1 \\ 0 & 2-\alpha \end{pmatrix}$$

$$= (2-\alpha)(\alpha^2 - 3\alpha + 1) - (2-\alpha) =$$

$$= 2\alpha^2 - 6\alpha + \cancel{2} - \alpha^3 + 3\alpha^2 - \cancel{\alpha} - \cancel{2} + \cancel{\alpha}$$

$$= -\alpha^3 + 5\alpha^2 - 6\alpha$$

$$= (-\alpha)(\alpha^2 - 5\alpha + 6)$$

$$= (-\alpha)(\alpha-2)(\alpha-3)$$

0 autoval.

2 "

3 "

b semidef. pentru

Trovare w t.c. $b(w,w)=0$

$w=(a,b,c)$ $\quad (a,b,c)\begin{pmatrix}2&1&0\\1&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=0$

$W \in$ autosp. di 0 $\quad \begin{pmatrix}2&1&0\\1&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

$\begin{pmatrix}2&1&0\\1&1&1\\0&1&2\end{pmatrix}$ $R_2 \to 2R_2-R_1$ $\begin{pmatrix}2&1&0\\\boxed{0}&1&2\\\boxed{0}&1&2\end{pmatrix}$ $R_3 \to R_3-R_2$ $\begin{pmatrix}2&1&0\\0&1&2\\0&0&0\end{pmatrix}$

$b=-2c$

$2a+b=0 \quad 2a-2c=0 \quad a=c$

soluz. $=\{(c,-2c,c) : c\in\mathbb{R}\} = L(1,-2,1)$

posso prendere $w=(1,-2,1)$ $\quad w\in Rad(b)$ $\quad b(w,w)=0$

Trovare w t.c. $b(w,w) > 0$

w autovett cioè $\alpha = 2$

$$\begin{pmatrix} 2-\alpha & 1 & 0 \\ 1 & 1-\alpha & 1 \\ 0 & 1 & 2-\alpha \end{pmatrix}$$

$\alpha = 2$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$b = 0$

$a = -c$

$L(-1,0,1)$

$w = (-1,0,1) \Rightarrow b(w,w) > 0$

- ~~simili diag.~~ M, M' simili hanno lo stesso polinomio caratt.

M diag. $\Rightarrow$ M è simile a $\begin{pmatrix} a_{11} & & & 0 \\ & a_{22} & & \\ & & \ddots & \\ 0 & & & a_{nn} \end{pmatrix} = M'$ $a_{11} \dots a_{nn}$ autoval.

$\det M' = a_{11}a_{22} \cdots a_{nn} = \prod \text{autoval.} = \text{termine noto del polinom. caratt.}$

$\boxed{\det M = \prod \text{autoval.}} = \qquad ''$

$\text{traccia } M' = a_{11} + a_{22} + \dots + a_{nn} = \sum \text{autoval.} = \text{coeff. di } \lambda^{n-1} \text{ nel polinom. caratt.}$

$\text{traccia } M = \sum \text{autoval.} = \qquad ''$

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2-\alpha & 1 & 0 \\ 1 & 1-\alpha & 1 \\ 0 & 1 & 2-\alpha \end{pmatrix} \qquad -\alpha^3 + 5\alpha^2 - 6\alpha + 0$$

diag.

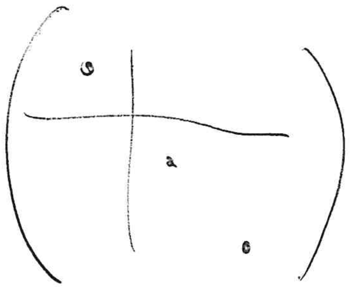
$$\underbrace{(2-\alpha)(1-\alpha)(2-\alpha)} + \dots$$

$n \times n$

$$(-1)^n \alpha^n + (\quad)\,\alpha^{n-1} + \text{termine noto}$$

$$(\quad) \downarrow (-1)^{n-1}(\text{somma degli elem. della diagonale}) \Downarrow \text{traccia}$$

termine noto $\downarrow$ det



$$\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \qquad \begin{pmatrix} 1 & 5 \\ 5 & 2 \end{pmatrix}$$

traccia \ det	+	−
+	$+ +$	$+ > -$
−	$- -$	$- > +$

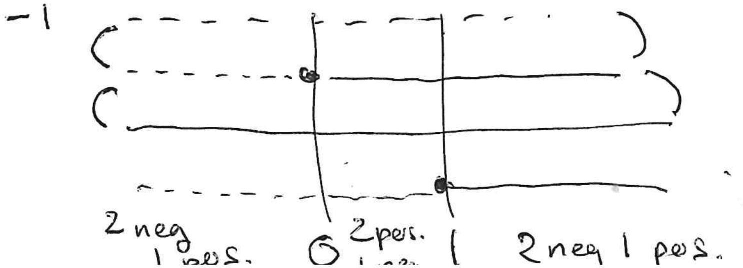
Ex Data $\begin{pmatrix} k & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ determ. k per cui è semidef. neg.

$$\det \begin{pmatrix} k-\alpha & 1 & 1 \\ 1 & -\alpha & 2 \\ 1 & 2 & -\alpha \end{pmatrix} = (k-\alpha) \det \begin{pmatrix} -\alpha & 2 \\ 2 & -\alpha \end{pmatrix} - \det \begin{pmatrix} 1 & 2 \\ 1 & -\alpha \end{pmatrix} + \det \begin{pmatrix} 1 & -\alpha \\ 1 & 2 \end{pmatrix}$$

$$= (k-\alpha)(\alpha^2 - 4) - (-\alpha - 2) + (2+\alpha) = -\alpha^3 + k\alpha^2 - 4k + 4\alpha + 4 + 2\alpha$$

$$= -\alpha^3 + k\alpha^2 + 6\alpha - 4k + 4$$

non $\exists$ valori di $k \neq 0,1$ per cui b è semi-def. neg. $\Leftarrow$



$k=0$ $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ $-\alpha^3+6\alpha+4$ Σ autoval. $= 0$ 0 non è autov.

$\Rightarrow$ ce ne sono pos. e neg.

$\Rightarrow$ b non è semidef neg.

$k=1$ $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ $-\alpha^3+\alpha^2+6\alpha$

$(-\alpha)\,(\underbrace{\alpha^2-\alpha}_{+}\underbrace{-6}_{-})$

0 autoval

autoval +

" −

b non è semidef neg.

Oss $\begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix}$ la forma bilineare

$\rightarrow$ traccia è 0

3 autoval nulli

$\swarrow$

$b=0$

c'è un autoval +

" " −

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & k & 1 \\ k & \bullet & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad \det \begin{pmatrix} 1-\alpha & k & 1 \\ k & 1-\alpha & 0 \\ 1 & 0 & 1-\alpha \end{pmatrix} = \det \begin{pmatrix} k & 1 \\ 1-\alpha & 0 \end{pmatrix} + (1-\alpha) \det \begin{pmatrix} 1-\alpha & k \\ k & 1-\alpha \end{pmatrix}$$

$$= (\alpha - 1) + (1-\alpha) \cancel{\det} (\alpha^2 - 2\alpha + 1 - k^2) =$$

$$(1-\alpha)\left[-1 + \alpha^2 - 2\alpha + 1 - k^2\right] = (1-\alpha)\left[\alpha^2 - 2\alpha - k^2\right]$$

$\alpha = 1$

$+ \qquad \leq 0$

Se $k \neq 0$ non è semidef.

$k = 0$ $\qquad (1-\alpha)\alpha(\alpha-2)$ $\quad 0, \; 1, \; 2$ $\qquad$ semidef $+$

Determ. $\forall k$ una base di $e_1^{\perp_b} = \{ \underset{(\alpha,\beta,\gamma)}{w} : b(e_1, w) = 0 \}$

$$b(e_1,w) = (1\ 0\ 0)\begin{pmatrix} 1 & k & 1 \\ k & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$(1\ k\ 1)\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \qquad \alpha + k\beta + \gamma = 0$$

$$\gamma = -\alpha - k\beta$$

$$e_1^{\perp_b} = \{ (\alpha, \beta, -\alpha - k\beta) : \alpha, \beta \in \mathbb{R} \} = L((1,0,-1),(0,1,-k))$$

$$(\alpha, \beta, -\alpha - k\beta) = \alpha(1,0,-1) + \beta(0,1,-k)$$

$$\underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -k \end{pmatrix}}_{\text{rango } 2} \Rightarrow \text{sempre lin. indip.}$$