Soluzione

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & k \end{pmatrix} = M$$

a)

b degenere $\Leftrightarrow$ $\det M = 0$

$\det M = k - 1$

$k = 1 \Leftrightarrow b$ degenere

b) $\mathrm{Rad}\, b$
- $k \neq 1$ $\underline{\mathrm{Rad}\, b = (0)}$ base $\emptyset$
- $k = 1$

$\mathrm{Rad}\, b$ = soluz $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} \end{array}\right)$ $x = -z$, $y = 0$

$\Vert$

$\{(-z, 0, z) : z \in \mathbb{R}\} = L(-1, 0, 1)$

base $\{(-1, 0, 1)\}$

$k = 2$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ \hline 1 & 0 & 2 \end{array}\right)$$

c) + d) Trovare una <u>base ortonorm.</u>

$\mathcal{B} = \{e_1, e_2, e_3\}$

$$u_3 = e_3 - b(e_3, e_1)e_1 - b(e_3, e_2)e_2$$
$$= e_3 - e_1$$

$$b(u_3, u_3) = b(e_3 - e_1, e_3 - e_1) =$$
$$\underbrace{b(e_3, e_3)}_{2} - 2\underbrace{b(e_3, e_1)}_{1} + \underbrace{b(e_1, e_1)}_{1} = 1$$

$\mathcal{B}_0$ base ortonormale $= \{e_1, e_2, e_3 - e_1)$

$\Rightarrow$ b è def. posit.

e) base di $L(e_3)^{\perp_b}$

$$L(e_3)^{\perp_b} = \left\{ (a,b,c) : (a,b,c) \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0 \right\}$$

$$\Updownarrow$$

$$(a+c,\ b,\ a+2c) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\Updownarrow$$

$$a+2c=0$$

$$L(e_3)^{\perp_b} = \{(-2c, b, c) : b, c \in \mathbb{R}\}$$

$$= L(\underbrace{(-2,0,1),(0,1,0)}_{\text{lin. indip}})$$

$$\Leftarrow \begin{pmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ ridotta}$$

$$\Downarrow$$

base $((-2,0,1),(0,1,0))$

f) Triangolo

$$\underset{O}{(0,0,0)} \qquad \underset{e_1}{(1,0,0)} \qquad \underset{u_3}{(-1,0,1)}$$

2 Lati ℓ_1 $e_1 - 0 = e_1$

ℓ_2 $u_3 - 0 = u_3$

$e_1 \perp_b u_3 \Rightarrow$ triangolo rett.

Area $\frac{1}{2} \|\ell_1\|_b \|\ell_2\|_b = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$

$\|\ell_1\|_b = \|e_1\|_b = 1 = \|u_3\|_b$

9) $(a,b,c) \in \mathbb{C}^3$ $(a,b,c) = v \neq 0$

$b((a,b,c),(a,b,c)) = 0$

$$(a,b,c)\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = (a+c, b, a+2c)\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$= a^2 + ac + b^2 + ac + 2c^2$

$a = 0$ $b^2 + 2c^2 = 0$ $b = \pm\sqrt{-2c^2}$

$b = \sqrt{2}$ $c = i$

$v = (0, \sqrt{2}, i)$

o

o

o

o

o

c