Esercizio 3

When $v \in \mathbb{R}^2$ and $M \in \mathbb{R}^{2,2}$ define $v \otimes M = T$ where

$$T_{ijk} = v_i M_{jk}$$

① Compute the products

$(1,2) \otimes \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\qquad$ $(2,1) \otimes \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

② Find a formula for the rank of $v \otimes M$

③ (Discussion.) Find a tensor which represents $v \otimes w^t \qquad \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^{2,2}$ and discuss the rank.