Correzione prova intermedia

$$b \longleftrightarrow M = \begin{pmatrix} 1 & k & 0 \\ k & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

ⓐ

$$\det \begin{pmatrix} 1-\alpha & k & 0 \\ k & 1-\alpha & 1 \\ 0 & 1 & 1-\alpha \end{pmatrix} = \underset{\alpha = 1}{(1-\alpha)} \left(\alpha^2 \overset{\text{var.}}{\frown} -2\alpha \overset{\substack{k\neq 0 \\ \text{perm.}}}{\frown} -k^2 \right)$$

$k \neq 0$ autovalori discordi $\quad 0$ non è autovalore $\Rightarrow$ b non è definita o semi-definita

<u>$k=0$</u>

$$(1-\alpha)(\alpha^2 - 2\alpha)$$

$\alpha = 1 \qquad \alpha = 0 \quad \alpha = 2$

tutti ≥ 0 $\Rightarrow$ semi-def. positiva.

(b) vettori isotropi

(x_1, x_2, x_0) $\qquad x_1 = 0$

$$\begin{array}{cc} e_2 & e_3 \end{array} \qquad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad b(e_2 - e_3, e_2 - e_3) =$$

$$= b(e_2, e_2) - 2b(e_2, e_3) + b(e_3, e_3) = 0$$

$(0, 1, -1)$ è isotropo $\forall k$

(c) base di $(e_1 + e_2)^{\perp}$

$$(1\ 1\ 0)\begin{pmatrix} 1 & k & 0 \\ k & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

$$0 = (1+k, 1+k, 1)\begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad (1+k)a + (1+k)b + c = 0$$

$$c = -(1+k)a - (1+k)b$$

$$(e_1 + e_2)^{\perp} = \{ (a, b, -(1+k)a - (1+k)b) : a, b \in \mathbb{R} \}$$

$$\| \quad a(1, 0, -1-k) + b(0, 1, -1-k)$$

rango $\begin{pmatrix} 1 & 0 & -1-k \\ 0 & 1 & -1-k \end{pmatrix} = 2$ sempre $\qquad \{(1, 0, -1-k), (0, 1, -1-k)\}$ base

(d) base$^{K=1}$ ortogonale

$(e_1, e_2, e_3) \rightarrow$ Gram - Schmidt

$b(e_1, e_1) = 1$

$v_2 = e_2 - b(e_1, e_2)\, e_1 = e_2 - e_1$

$b(v_2, v_2) = 0 \Rightarrow$ problemi

(e_1, e_3, e_2)

$v_3 = e_3 - b(e_1, e_3)\, e_1 = e_3$ $\qquad b(e_3, e_3) = 1$

$v_2 = e_2 - b(e_1, e_2)\, e_1 - b(e_3, e_2)\, e_3 = e_2 - e_1 - e_3$

$(e_1, e_3, e_2 - e_1 - e_3)$ base ortog.