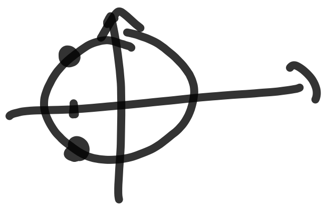
∘ 1) Find $X$ not simply conn.

$\sim$ equiv. relation $\Rightarrow$ $X/\sim$ is simply conn

$S^1$ $\quad \pi(S^1) = \mathbb{Z}$ $\quad x \sim y \quad \forall x, y \in S^1$

- $S^1/\sim = \{P\}$ is simply conn.
- Other possibilities



$$(x,y) \sim (x',y') \Rightarrow x = x'$$
$$\forall (x,y)(x',y') \in S^1$$

$S^1/\sim \; = [-1, 1]$ simpley connected because contrachible.

2) $Y \subseteq X$

A) $X$ contrachible $\quad Y$ is not

example $X = \mathbb{R}^2$ is contrachible $\quad Y = \mathbb{R}^2 - \{0\}$ not contrachble

$\pi(Y) = \mathbb{Z}$

B) $X$ is not contrachible $\quad Y$ is contrachble

$X = \mathbb{R}^2 - \{0\}$ $\quad Y = \{P\} \quad P \in \mathbb{R}^2 - \{0\}$

3) $f: X \to Y$ $g: Y \to X$
continuous

$Y$ connected $g \circ f \sim id_X$

$\Rightarrow X$ connected.

Assume $X_1, X_2$ is a disconnection of $X$

$X \xrightarrow{f} Y \xrightarrow{g} X$ $g(Y)$ is connected
say $g(Y) \subseteq X_1$

take $P \in X_2$ $H$ homotopy between $g \circ f$ and id

$H: X \times [0,1] \to X$ $H$ continuous

$H(P,0) = (g \circ f)(P) \in X_1$
$H(P,1) = id(P) \in X_2$

$P \times [0,1] \subseteq X \times [0,1]$ homeom. to $[0,1]$
connected

$H(P \times [0,1])$ connected $\Rightarrow$

either $H(P \times [0,1]) \subseteq X_1$ NO since $H(P,1) \in X_2$
or $H(P \times [0,1]) \subseteq X_2$ NO since $H(P,0) \in X_1$

a CONTRADICTION

$\Rightarrow$ there are no disconnections of $X$