

Advanced Geometry 2024/2025
Intermediate Exercises

Exercise 1. -

Find an example of a topological space X and an equivalence relation \sim on X such that X/\sim is simply connected while X is NOT simply connected.

Exercise 2. -

Let X be a topological space and $Y \subset X$ a subset, endowed with the induced topology.

a) Find an example in which X is contractible while Y is NOT contractible.

b) Find an example in which Y is contractible while X is NOT contractible.

Assume then that $Y \not\subset X$. Find an example in which both X, Y are contractible while $X \cup Y$ is NOT contractible.

Exercise 3. -

Let X, Y be topological spaces and let $f : X \rightarrow Y$ and $g : Y \rightarrow X$ be continuous functions. Show that if Y is connected and the composition $g \circ f$ is homotopic to the identity, then X is connected.