Correction

X, Y simply conn. closed $\subseteq \mathbb{R}^2$

Case 1 $X \cap Y = \{P\}$

$X - \{P\}$ $\quad Y - \{P\}$ connected compon. of $(X \cup Y) - \{P\}$

$\quad \shortparallel \qquad\qquad \shortparallel$

$(X \cup Y) \cap (\mathbb{R}^2 - Y) \quad (X \cup Y) \cap (\mathbb{R}^2 - X)$

disconnection of $(X \cup Y) - \{P\}$

$X \cup Y$ simply connected

$\alpha : [0,1] \longrightarrow X \cup Y \qquad \alpha(0) = \alpha(1) = P$

$\alpha^{-1}(P) = $ closed in $[0,1]$

$[0,1] - \alpha^{-1}(P)$ open hence a union of intervals

$\cup\!|$

max (t_1, t_2) connected $\qquad \alpha(t_1, t_2) \subseteq X$ or Y

$\subseteq X \cup Y - \{P\}$

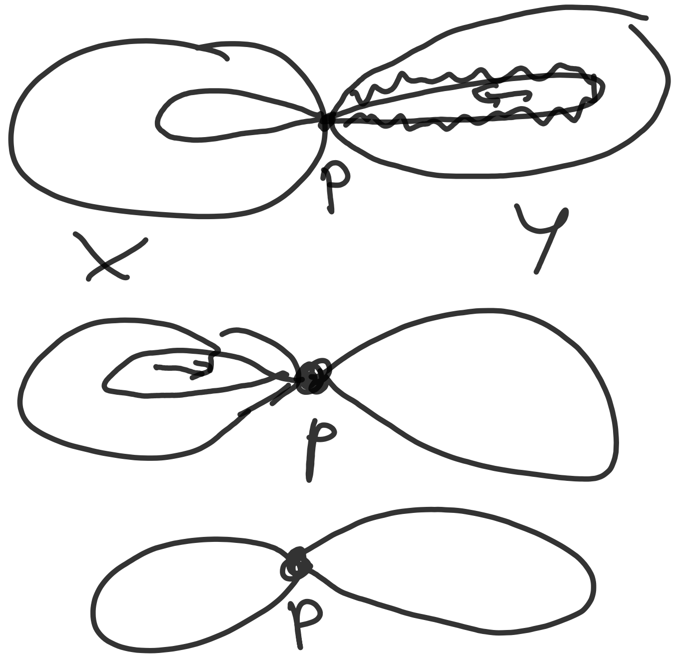
$\alpha(t_1) = \alpha(t_2) = P$ for maximality $\longrightarrow \alpha[t_1, t_2] \subseteq X$ or Y

$\alpha|_{[t_1, t_2]} = $ loop based at P $\subseteq X$ or $\subseteq Y$ $\sim$ constant loop

So I can replace α with a loop $\bar{\alpha} \sim \alpha$ all contained in X just by substituting $\alpha|_{[t_1, t_2]}$ with the constant loop if $\alpha|_{[t_1, t_2]} \subseteq Y$

Since X is simply conn. $\Rightarrow \bar{\alpha} \sim$ constant

$\Rightarrow \alpha \sim$ constant

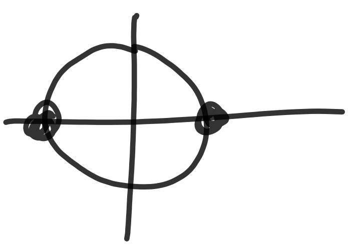


NB If X, Y both open in $X \cup Y$ $\Rightarrow$ since $X \cap Y = \{p\}$ pathwise conn.

$\Downarrow$

$X \cup Y$ simply conn.

CASE 2 $X \cup Y$ can be non-simply conn.



$X = S^1 \cap \{y \geq 0\}$ $\quad X \cap Y = 2$ pts

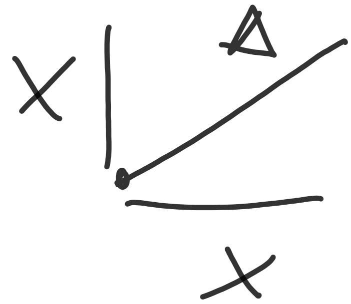
$Y = S^1 \cap \{y \leq 0\}$

X, Y homeom. to $[-1,1]$ simply conn.

$X \cup Y = S^1$ not simply conn.

Ex 2

$X \times X \supseteq \Delta = \{(x,x) : x \in X\}$



X pathwise conn.

$\Longrightarrow \Delta$ pathwise conn.

$\forall \; P = (x,x) \in \Delta$
$Q = (y,y) \in \Delta$

$x, y \in X \quad \exists \; \alpha : [0,1] \to X \quad \alpha(0) = x, \; \alpha(1) = y$

$\bar{\alpha}(t) = (\alpha(t), \alpha(t)) : [0,1] \to \Delta \subseteq X \times X$

$\bar{\alpha}(0) = P \quad \bar{\alpha}(1) = Q$

$\bar{\alpha} = \alpha \times \alpha$ so it is continuous

NB Δ is homeom to X

$f : \Delta \to X = \pi_1$ (continuous)

$g : X \to \Delta \quad g(x) = (x,x)$

$g \circ f = 1_\Delta \quad f \circ g = 1_X$

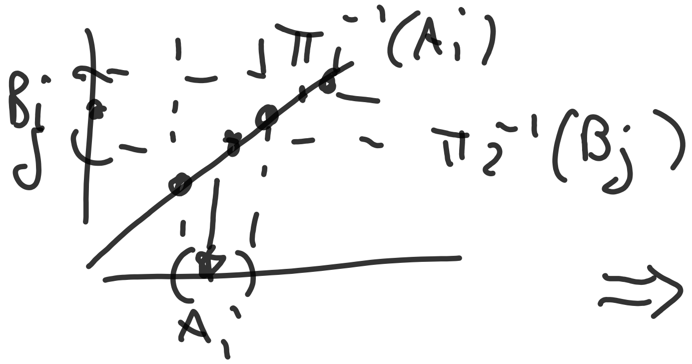
An open set in Δ is

$\Delta \cap \left(\bigcup \pi_1^{-1}(A_i) \cap \pi_2^{-1}(B_j) \right) = U$

$g^{-1}(U) = \underline{g^{-1}(\Delta)} \cap \left(\bigcup g^{-1}\pi_1^{-1}(A_i) \cap g^{-1}\pi_2^{-1}(B_j) \right)$

$$g^{-1}\pi_1^{-1}(A_i) \qquad\qquad g^{-1}\pi_2^{-1}(B_j)$$

$$g^{-1}\{(x,x) : x \in A_i\} = \{x : x \in A_i\} = A_i \qquad\qquad = B_j$$



$$g^{-1}(U) = \underbrace{\bigcup (A_i \cap B_j)}_{\text{open in } X}$$

$\Rightarrow g$ continuous

<u>Ex 3</u> $Y \subseteq X$ $\quad X$ simply conn.

$f : X \to Y \quad \underbrace{f|_Y = id_Y}$

$\pi(f) : \underset{(e)}{\pi(X)} \to \pi(Y)$ $\qquad j : Y \to X$ inclusion

$$Y \xrightarrow{j} X \xrightarrow{f} Y \qquad \underline{f \circ j = f|_Y}$$

$$\underbrace{\pi(f \circ j)}_{= \pi(f) \circ \pi(j)} = \pi(id_Y) = \underline{id_{\pi(Y)}}$$

$$\pi(Y) \xrightarrow{\pi(j)} \underset{(e)}{\pi(X)} \xrightarrow{\pi(f)} \pi(Y)$$

$\forall \alpha \in \pi(Y)$

$$\pi(f) \circ \pi(j)(\alpha) = \pi(f)(e) = e$$

$$\pi(f \circ j)(\alpha) = id_{\pi(Y)}(\alpha) = \alpha \quad \Rightarrow \quad \alpha = e \Rightarrow \pi(Y) = (e)$$