Advanced Geometry 2023/24.

Intermediate assessment

**Exercise 1.** -

*Let $X, Y$ be simply connected closed subsets of $\mathbb{R}^2$. Determine if $X \cup Y$ is necessarily simply connected in the following two cases (by giving an argument if the answer is positive or a counterexample if the answer is negative).*

*Case 1.* $X \cap Y =$ *a singleton.*

*Case 2.* $X \cap Y =$ *2 points.*

**Exercise 2.** -

*Prove that if $X$ is pathwise connected, in the product $X \times X$ the subset $\Delta = \{(x, x) : x \in X\}$ (also called the* diagonal*) is pathwise connected.*

**Exercise 3.** -

*Let $X$ be a simply connected topological space and take a pathwise connected subset $Y \subset X$, with the induced topology. Assume there exists a continuous map $f : X \to Y$ such that the restriction $f_{|Y}$ is the identity on $Y$. Prove that $Y$ is simply connected.*