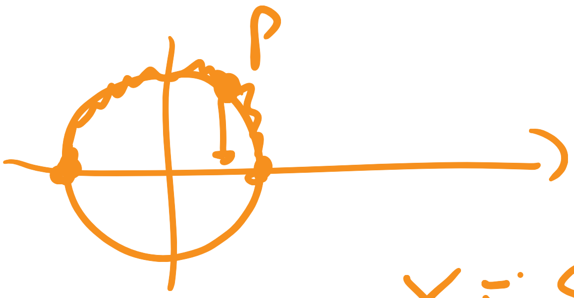
Ex 1



$X = S^1 \cap \{y \geq 0\}$

$Y = S^1 \cap \{y \leq 0\}$

$X \cup Y = S^1$ not simply conn.

$X \cap Y = \{(1,0), (-1,0)\}$ not even connected

X, Y are both simply conn. because both are homeom. to $[-1,1]$

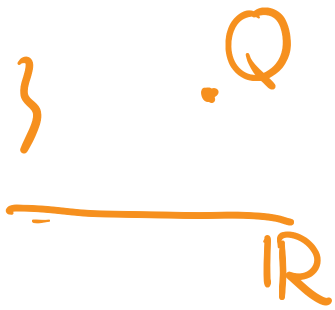
e.g. $X \to [-1,1]$ $\quad p = (x,y) \in X \mapsto x \in [-1,1]$

$[-1,1] \to X$ $\quad t \in [-1,1]$ $\quad t \mapsto (t, \sqrt{1-t^2})$

$[-1,1]$ is simply conn. $\Rightarrow$ X is simply conn.

Ex 2

$X = S^1 \ni p$ $\qquad Y = \mathbb{R} \cup \{Q\}$



$X \not\simeq Y$ because X conn.
Y not conn.

$X - \{p\} = S^1 - \{p\}$ homeom. to $\mathbb{R}$

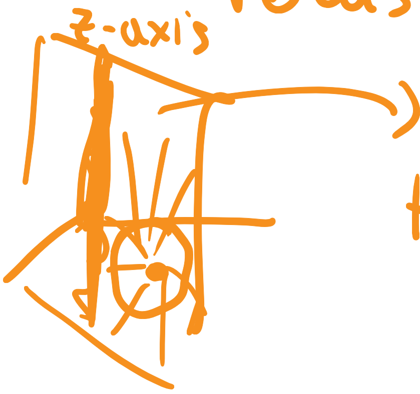
$Y - \{p\} = \mathbb{R}$

Ex 3

$\mathbb{R}^3 - \{z \text{ axis}\} - \{\text{circle } S\}$

$\simeq$

torus.



$H \simeq \mathbb{R}^2$

$H - \{p\} \simeq \mathbb{R}^2 - \{p\}$ homeom. $\simeq$ homotop S^1

which holds on each intersection of $\mathbb{R}^3$ - circle - z axis with a plane through the z-axis.

This homotopy generalises to a homotopy of $\mathbb{R}^3 - Z \to$ torus.

<u>Rem</u> Both $\mathbb{R}^3 - Z$ and torus have $\pi = \mathbb{Z} \times \mathbb{Z}$

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