Advanced Geometry 2021/22
Intermediate Exercises

Exercise 1. -

Show that if X, Y are both simply connected and their intersection is non-empty, yet $X \cap Y$ and $X \cup Y$ can be not simply connected

Exercise 2. -

Find topological spaces X, Y, and points $P \in X$, and $Q \in Y$, such that X is not homotopy equivalent to Y, but $X - \{P\}$ is homotopy equivalent to $Y - \{Q\}$.

Exercise 3. -

Let $Z \subset \mathbb{R}^3$ be the set defined as

$$Z = \{circle\ x^2 + y^2 = 1\} \cup \{z\ axis\}.$$

Show that $\mathbb{R}^3 - Z$ is homotopy equivalent to a torus.