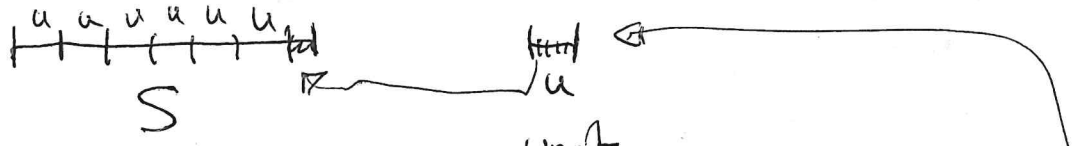
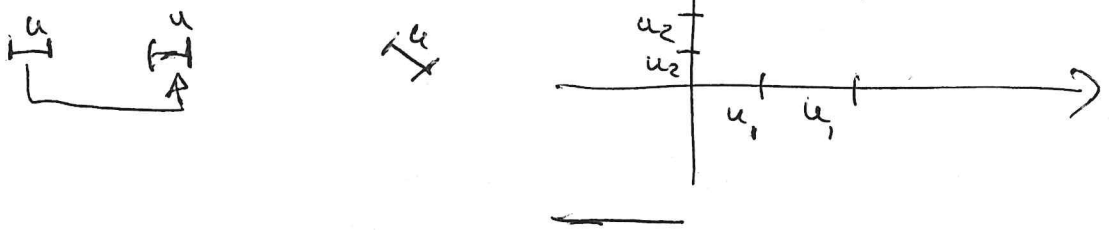
Geometria

- measure (primary school)
- descriptive geometry (secondary school)
- analytic geometry (high school)



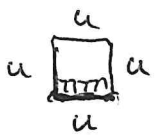
2 aspects 1) no overlapping

2) need notion of "same length"
stessa lunghezza

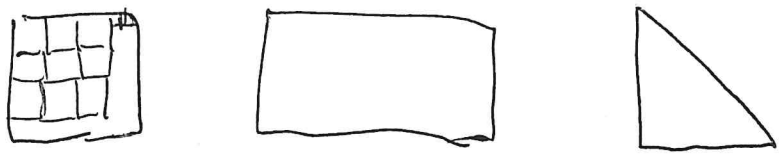


per la misura delle aree serve un'unità di dim. 2

- in order to measure areas I need a unit of dim. 2



dal segmento al quadrato
segment to square.

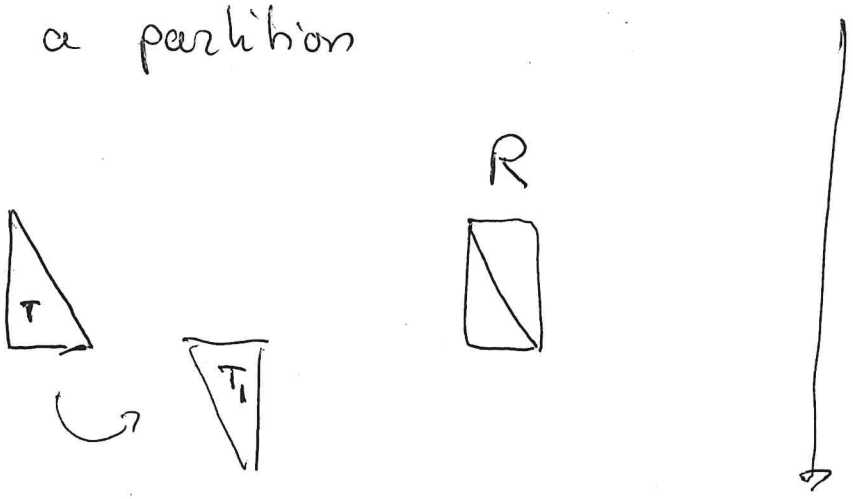


square rectangle ?

Bisogna usare le partizioni
Need to use partitions

Need a fundamental principle principio fondamentale

L'area di un oggetto è la somma delle aree
The area of a geometric object (of dim 2)
degli elementi di una partizione
is the sum of the areas of elements
of a partition



area of T + area of T_1 = area of R

||

area of T + area of T

||

2 area of T

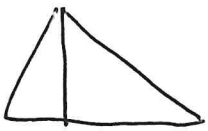
$$T = \frac{1}{2} \text{ area of } R.$$

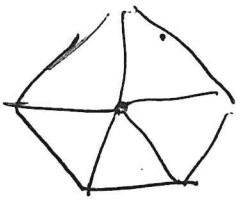
Area of rectangles ⇒ area of triangles

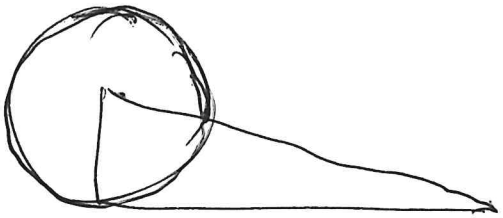
⇓

Area of objects which decompose in triangles

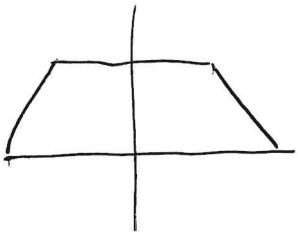
aree degli oggetti decomponibili in triangoli

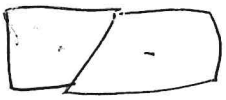


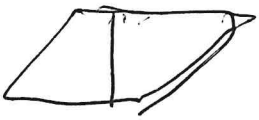




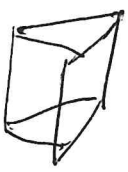
πR^2







$\frac{1}{3}$



Due aspetti
Two aspects

(1) In Euclid's Element

- The whole is equal to the sum of its parts

L'intero è uguale alla somma delle parti

COULD BE MISLEADING for 2 reasons
PUÒ FUORVIARE

- parts must be non-overCapping
  Le parti non si devono sovrapporre

Area of $T_1 \cup T_2$ = area of T_1 + area of T_2 − area of $T_1 \cap T_2$

- intero ⇔ area dell'intero
  whole ⇔ the area of the whole

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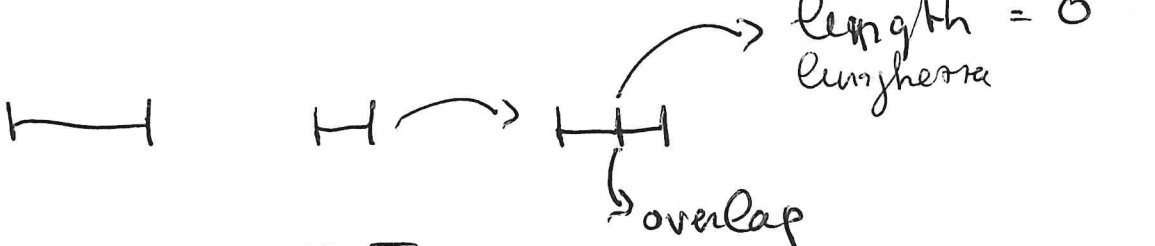
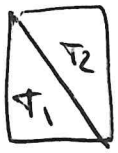
(2) "Non overlapping" in the overlap



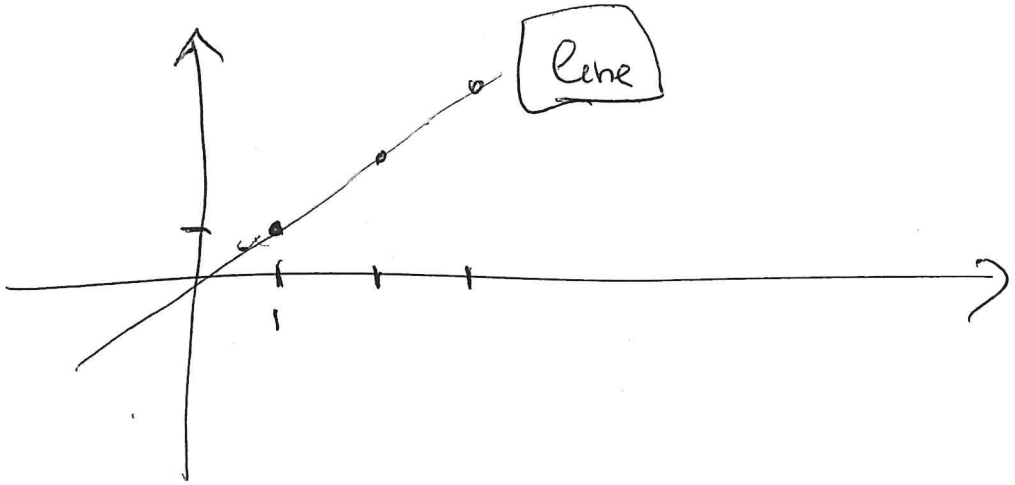


T_1, T_2
They overlap
in effetti si sovrappongono,

area = 0

no

Coord. oriented length Lunghezza orientata



analytic geometry

piano
plane $\Leftrightarrow$ $\mathbb{R} \times \mathbb{R}$

geometry $\Rightarrow$ study of some relations
studio di alcune relazioni

choose $\frac{a}{b}$ $\quad x R y \Leftrightarrow \frac{x}{y} = \frac{a}{b}$

$x : y = a : b$

rappresentare la relazione
draw the relation i.e.

draw the points (x, y) s.t. $x R y$

ex $\frac{a}{b} = \frac{2}{3}$

$x = 1 \quad y = 3/2$

$x = 2 \quad y = 3$

$x = 3 \quad y = 9/2$

Line

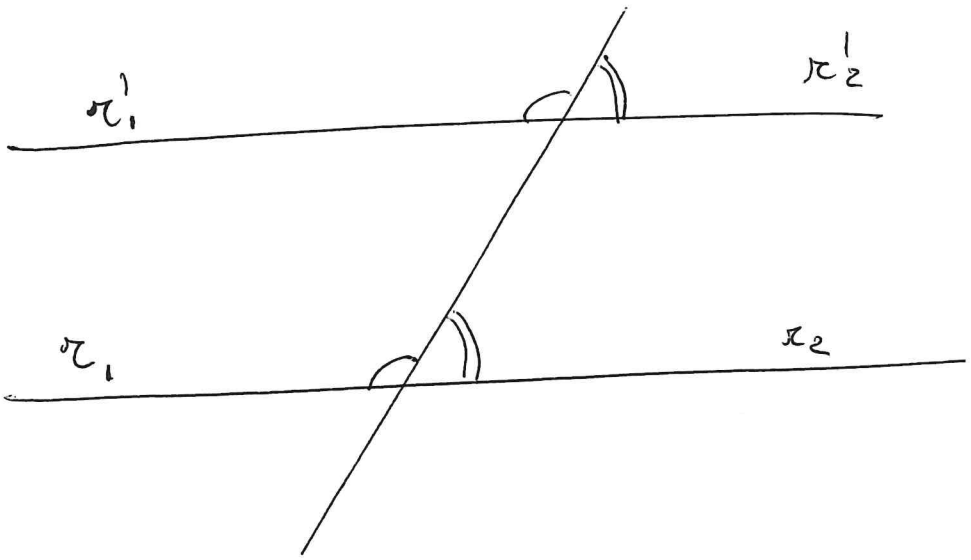
$\frac{b}{a} = m$

$y = mx$

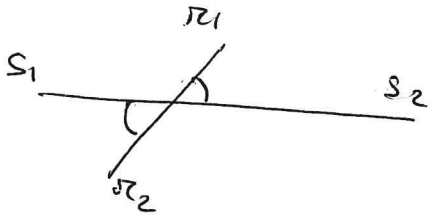
Assiomi di Euclide

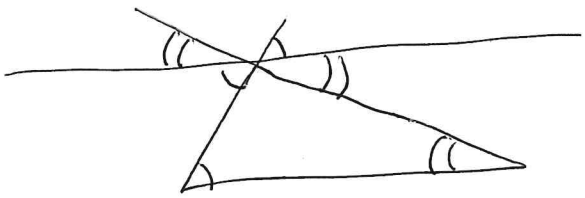
1) Per due punti passa una retta
2) Le rette sono estendibili indefinitamente
3) Per ogni P e $r>0$ esiste un cerchio di centro P e raggio r
4) Tutti gli angoli retti sono congruenti
5) Postulato delle parallele
Per un punto $P \notin r$ si può tracciare una e una sola retta parallela a r

1) Through two points one can draw one line
2) Every line can be extended indefinitely
3) For all points P and $r>0$ there exists a circle with center P and radius r
4) All right angles are congruent
5) For all $P \notin r$ one can draw a unique line parallel to r



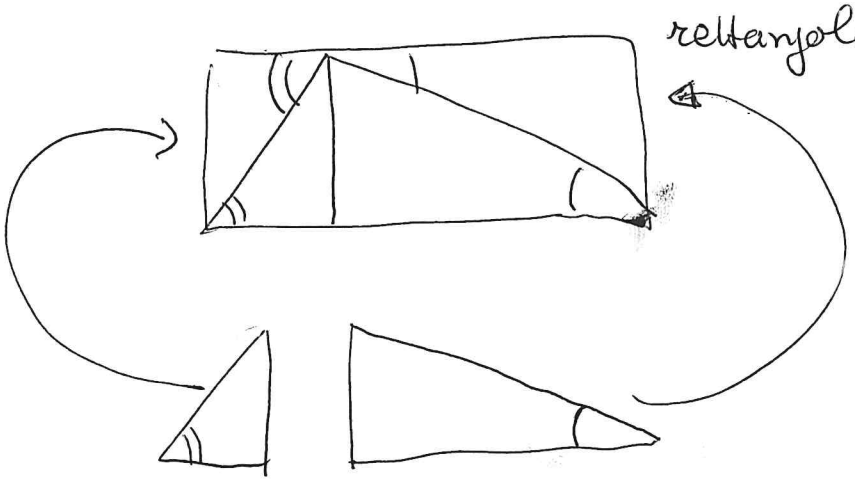
r_1, r_2 semirette half-lines



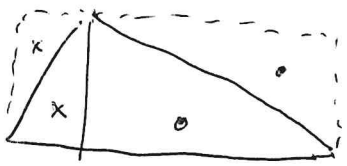


Somma = 180°

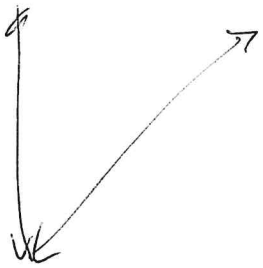
The sum of internal angles is 180°

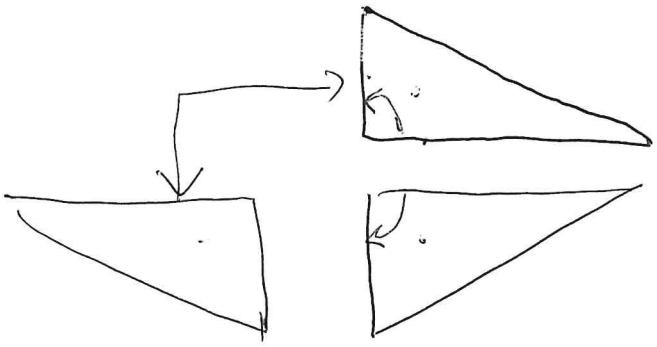


Programma di Erlangen



uguale ⇔ congruente





uguale ?