Stochastic Energy Pricing of an Electric Vehicle Parking Lot

Giovanni Gino Zanvettor*, Marco Casini*, Roy S. Smith**, Antonio Vicino*

***Abstract*—The increasing adoption of electric vehicles (EVs) and the related need for efficient battery charging leads to additional challenges to the power network and energy providers. One of the main issues regards the intrinsic uncertainty affecting the EV charging process, which calls for appropriate strategies to ensure reliable solutions. From the perspective of a parking lot, the electric vehicle charging process should be managed in order to guarantee the recharge at competitive price. In this paper, the problem of energy pricing under vehicle uncertainty is addressed. Specifically, we propose a new energy pricing strategy where the daily profit of the parking lot is guaranteed with a given probability level. The source of uncertainty is related to the EV arrival/departure times and the daily number of incoming vehicles. By exploiting photovoltaic (PV) and electrical storage system (ESS) facilities, procedures and algorithms are formulated to compute the optimal selling price and to operate the battery in a receding horizon framework. Since the proposed chance constraint problems are intractable for realistic scenarios, suitable approximations are provided in order to find a feasible solution. Numerical results show the effectiveness of the proposed approach and the tightness of the introduced relaxation, even in the presence of a high number of incoming vehicles.**

***Index Terms*—Electric vehicles, charging stations, pricing, stochastic optimization, receding horizon.**

Nomenclature

Δ	Sampling time
P_0^{EV}	Nominal EV charging power
t_v^a	Arrival time (plug-in time) of the v-th EV
t_v^d	Departure time (plug-out time) of the v-th EV
t_v^c	Charging time of the v-th EV
N_V	Total number of daily vehicles
$H(t)$	Number of vehicles which are charging at time t
$H(\tau,t)$	Number of connected vehicles at time τ which will be still charging at time t
$E^{ESS}(t)$	Battery level of charge at time t
$P^{ESS+}(t)$	Battery charging power in time slot t
$P^{ESS-}(t)$	Battery discharging power in time slot t
$P^{ESS}(t)$	Net power exchanged into the battery in time slot t
$E^{EV}(t)$	Energy required to charge plugged-in EVs in time slot t
$\widehat{E}^{PV}(t)$	Energy drained from the PV plant in time slot t
$E^{PV}(t)$	Energy production of the PV plant in time slot t
$E^{G}(t)$	Energy drawn from the grid in time slot t
$p(t)$	Grid electricity price in time slot t
$c(t)$	Electricity cost in time slot t
η	ESS charging/discharging efficiency
α	Fraction of daily profit with respect to daily cost
s	Daily electricity selling price
ε	Probability level
β	Auxiliary probability level

* Dipartimento di Ingegneria dell'Informazione e Scienze Matematiche, Università di Siena, Via Roma 56, 53100 Siena, Italy. E-mail: {casini,vicino,zanvettor}@diism.unisi.it
** Automatic Control Lab, ETH, Zürich 8092, Switzerland, E-mail: rsmith@control.ee.ethz.ch

I. Introduction

To overcome issues caused by pollution and CO_2 emissions, electric vehicles (EVs) are becoming an appealing opportunity, especially in large cities. In fact, EVs make it possible to move CO_2 emissions outside living centers. Moreover, in contrast to from internal combustion engine vehicles, they can exploit renewables (e.g., solar, wind, etc.) for their charging. However, EVs may be source of different critical issues for safe and reliable power grid operation while ensuring user satisfaction/comfort. On the power side, attention has been focused on several aspects involving cost minimization approaches and peak shaving targets. In [1], a battery swapping strategy is proposed to satisfy safety power grid constraints while minimizing the grid operation cost. In [2], [3], the problem of sizing and siting of charging stations is studied to provide a proper charging service. Charging unit management inside an airport EV parking lot is presented in [4]. In [5], game theory is exploited to derive a smart energy management of an EV parking lot. To improve the advantages given by renewable generation and electricity storage facilities, in [6] a smart charging management system for an EV parking lot is developed, while in [7] a parking lot energy management strategy is designed by considering EV mobility and parking patterns. In [8], a receding horizon strategy to minimize grid operation cost of an industrial microgrid is presented.

Paying attention to EV owner customer satisfaction, in [9], [10] battery swapping strategies are derived to optimize the charging station cost while ensuring a fast fueling service. In [11], a charging strategy is developed to reduce user electricity cost by employing incentive programs, whereas in [12] the same goal is achieved by considering user time anxiety.

As well known, EVs represents an intrinsic source of uncertainty due to traffic conditions and user preferences. To handle such problems, several solutions have been provided in the literature. In [13], the profit maximization of an EV parking lot under uncertain EV arrival and departure times is investigated. To manage the uncertain load of EVs, a grid cost minimization strategy is proposed in [14], while in [15] a peak power reduction problem is addressed.

From EV electricity pricing perspective, the investigated topics are focused on providing solutions about problems involving both user and power system side. In [16], a game theory study is carried out in order to derive an optimal selling price for several charging stations. Considering power and transportation network frameworks, a pricing strategy to optimally manage a large number of EVs is developed in [17]. To reduce the overlap between residential and EV charging station loads, in [18] a dynamic pricing procedure aimed at shifting the EV demand to low peak hours is reported. In [19], a two layer model is developed to generate electricity selling price under market price uncertainty, while in [20] an EV energy pricing procedure to lower the peak power absorption is proposed.

To handle the uncertainty on EVs and renewables, an electricity pricing framework is developed in [21]. In [22], a pricing mechanism to fill load valleys in the presence of EVs is designed. In such a strategy, the recharge of vehicles is considered for both cooperative and non-cooperative scenarios. A pricing strategy based on EV user responses to different prices is reported in [23]. In particular, assuming that the utility company may individually adapt the energy price, a profit maximization technique is proposed. In [24], a distribution locational marginal pricing method to mitigate system congestion caused by uncontrolled charging of EVs is introduced. A charging station providing vehicle-to-grid services is considered in [25]. In this scenario, the pricing problem is analyzed by considering battery degradation and by proposing different charging contracts to users. In [26], a real-time energy pricing strategy based on the concept of inverse-demand function under renewable and load demand uncertainty is proposed. To provide a probabilistic guarantee for the EV charging profit, a chance constrained program under EV load uncertainty is formulated in [27].

Paper Contribution

In this work, a parking lot equipped with a charging station, photovoltaic (PV) generation and electrical energy storage system (ESS) is considered. The main contribution of the paper is twofold:

i) To design a procedure aimed at computing the daily selling price for charging EVs. In particular, such a price must be the lowest which ensures a given profit with a probabilistic guarantee. Moreover, the selling price is considered to be constant throughout the day for commercial reasons. In the considered setting, the source of uncertainty is related to future incoming vehicles, specifically, to arrival time, departure time and the daily number of incoming EVs. To handle such uncertainties, a chance constrained optimization problem aimed at finding the optimal selling price is designed. To find a tractable formulation of such problem, the optimized Bonferroni approximation described in [28] is exploited.

ii) To provide a receding horizon control algorithm to operate the ESS during the day to achieve the expected profit of i). Contrary to the procedure designed in i), this algorithm takes advantage of the knowledge acquired in an online approach about the random process outcomes of arrived vehicles.

The major novelty of the contribution is to provide a robust selling price policy as the solution of an optimization problem where the uncertainty on the EVs is modeled through suitable probability distributions. The proposed approach allows one to deal with different configurations and scenarios because no specific assumptions on probability distributions describing the EV uncertainties is made.

The proposed algorithms are based on the solution of mixed integer linear programs which are tractable for realistic applications, as witnessed by the extensive numerical simulations performed.

Paper Organization

The paper is organized as follows. In Section II, problem variables, constraints and probability derivations are described. In Section III, the optimization problem aimed at finding the optimal daily selling price is analyzed and solved through a suitable algorithm. Section IV is related to the formulation of a receding horizon procedure to operate the ESS during the day. In Section V, numerical results to evaluate the effectiveness and the computational feasibility of the proposed approach are reported. Finally, conclusions and future research lines are drawn in Section VI.

II. PROBLEM FORMULATION

In this work, we focus on a charging station used to charge a large number of electric vehicles. The charging station is assumed to be equipped with charging units, a PV plant and an electrical storage system. It is supposed that the number of charging units is enough to provide charging for all incoming vehicles, and so no vehicle queues are considered. Moreover, it is assumed that the daily electricity cost profile and a forecast of the PV daily production are available in advance. In the setting considered, the charging station is not allowed to sell energy to the grid, i.e. no vehicle-to-grid energy flow is considered.

The problem is formulated in a discrete time setting, where the sampling time is denoted by Δ. The EV arrival and departure times, and the daily number of incoming vehicles are uncertain and their probability distributions, or estimates for them, are assumed to be available. Each plugged-in vehicle is charged at a constant power rate P_0^{EV} from the arrival time to its departure time. We assume that the reference day is divided into T time slots where the initial time slot starts at $t = 0$, while the last one at $t = T-1$. Let t_v^a be the v-th vehicle arrival time and t_v^c the corresponding charging time, i.e. the number of time slots the vehicle remains in charge. Without loss of generality, we consider that vehicles leave the parking lot at time $t_v^d = t_v^a + t_v^c$. In fact, in a realistic scenario, vehicles may keep a charging unit busy longer than the charging duration. However, such vehicles can be neglected after t_v^d, since they are no longer charging and the number of charging units is assumed to be enough to receive incoming vehicles. The total number of incoming vehicles during each day is represented by the random variable N_V. Let us assume that the random variables related to arrival time and charging time of each vehicle are independent and identically distributed. Thus, the arrival time, departure time and charging time of a generic vehicle are denoted by t^a, t^d and t^c, respectively. Moreover, we assume that the supports of random variables t^c and N_V are bounded. We denote their minimum values by $\underline{t}^c$ and $\underline{N}_V$, and their maximum values by $\overline{t}^c$ and $\overline{N}_V$.

Let us define by $E^{EV}(t)$ the overall energy required to charge plugged in vehicles in the t-th time slot, i.e. between time t and $t+1$.

Concerning the ESS, we denote by $E^{ESS}(t)$ the stored energy at time t, and its charging/discharging power rates by $P^{ESS^+}(t)/P^{ESS^-}(t)$, respectively. We use a simple dynamic model for the ESS

$$E^{ESS}(t+1)=E^{ESS}(t)+\eta\Delta P^{ESS^+}(t)-\frac{1}{\eta}\Delta P^{ESS^-}(t),$$

where η is the battery efficiency. Let E_0^{ESS} denote the energy stored at the initial time step of the day, i.e.

$$E^{ESS}(0)=E_0^{ESS}.$$

The ESS charging and discharging power rates are bounded by zero and a maximum rate $\overline{P}^{ESS}$, i.e.

$$0\le P^{ESS^+}(t)\le \overline{P}^{ESS}$$
$$0\le P^{ESS^-}(t)\le \overline{P}^{ESS},$$

while the energy level is bounded by its capacity $\overline{E}^{ESS}$, i.e.

$$0\le E^{ESS}(t)\le \overline{E}^{ESS}.$$

Let us define the net power exchanged by the battery as

$$P^{ESS}(t)=P^{ESS^+}(t)-P^{ESS^-}(t).$$

Notice that, in order to have a consistent ESS dynamics, $P^{ESS+}(t)$ and $P^{ESS-}(t)$ cannot be both nonzero. Thus, the following constraint will be enforced

$$P^{ESS+}(t)P^{ESS-}(t)=0.$$

Let $E^{PV}(t)$ be the PV plant energy production between time t and $t+1$ and let $\widehat{E}^{PV}(t)$ denote the energy that is injected into the parking lot charging station, during the same time interval. Hence $\widehat{E}^{PV}(t)$ is such that:

$$0\le \widehat{E}^{PV}(t)\le E^{PV}(t).$$

Notice that, $\widehat{E}^{PV}(t)$ is different from $E^{PV}(t)$ when curtailment is necessary, i.e., when the PV production exceeds that required by the parking lot for managing EV charging and ESS operation.

Thus, the overall energy drawn from the grid $E^G(t)$ in time slot t is

$$\begin{aligned}E^G(t)&=\max\Big\{\underbrace{E^{EV}(t)-\widehat{E}^{PV}(t)+\Delta P^{ESS}(t)}_{\widehat{E}^G(t)},0\Big\}\\&=\max\Big\{\widehat{E}^G(t),0\Big\},\end{aligned}\tag{1}$$

where the max function is needed since the parking lot is not allowed to export energy to the grid.

Let us define the cost per time step $c(t)$ as

$$c(t)=E^G(t)p(t),\tag{2}$$

where $p(t)$ is the electricity price at time t.

The aim of this work is summarized in the following problem.

Problem 1. *For each day, compute the minimum electricity selling price s' such that the daily net profit is at least a fraction α of the daily cost, with probability at least $1-\varepsilon$.*

To fulfill Problem 1, the overall revenue must satisfy the following inequality with probability $1-\varepsilon$.

$$\sum_{t=0}^{T-1}E^{EV}(t)s'\ge(1+\alpha)\sum_{t=0}^{T-1}c(t).\tag{3}$$

By defining $s=s'/(1+\alpha)$, (3) is equivalent to

$$\sum_{t=0}^{T-1}E^{EV}(t)s\ge\sum_{t=0}^{T-1}c(t).$$

So, the goal is to find an *artificial* selling price s capable of covering the daily cost. Then, the *actual* selling price will be $s'=(1+\alpha)s$. Hereafter, we will focus on the computation of s which, for the sake of simplicity, will be denoted as the *selling price*.

To compute the probability distribution of the daily net profit, we focus on finding the distribution of the energy drawn from the grid at each time step. Since the randomness is due to vehicle statistics, let us focus on random variables modeling the EV process.

Denote by $\mathcal{P}(A)$ the probability that a given event A occurs. Let $\mathcal{P}(A,B)$ be the joint probability that events A and B occur, and $\mathcal{P}(A\cup B)$ be the probability of the union of the two events. The probability that an event A occurs conditioned on B is denoted by $\mathcal{P}(A|B)$. Moreover, let $\overline{A}$ be the complement of event A, i.e. $\mathcal{P}(\overline{A})=1-\mathcal{P}(A)$.

The probability that a generic vehicle is in charge at time t can be expressed as

$$\begin{aligned}&\mathcal{P}\left(t^a\le t,\ t^d>t\right)=\\&=\sum_{\tau=\max\{t-\overline{t}^c+1,0\}}^{t}\mathcal{P}\left(t^d>t|t^a=\tau\right)\mathcal{P}\left(t^a=\tau\right)\\&=\sum_{\tau=\max\{t-\overline{t}^c+1,0\}}^{t}\mathcal{P}\left(t^c>t-\tau\right)\mathcal{P}\left(t^a=\tau\right),\end{aligned}\tag{4}$$

that is the convolution between the complement of the charging time cumulative distribution with the distribution of the arrival time. Notice that, the probability of having a vehicle in charge at time t follows a Bernoulli distribution, so we can easily compute the probability that a given amount of vehicles be in charge at a given time step. In fact, let $H(t)$ be the random variable denoting the number of vehicles in charge at time t. Then, its probability distribution is derived as follows

$$\mathcal{P}(H(t)=n)=\sum_{m=n}^{\overline{N}_V}\mathcal{P}(H(t)=n|N_V=m)\,\mathcal{P}(N_V=m)\tag{5}$$

where

$$\begin{aligned}&\mathcal{P}\left(H(t)=n|N_V=m\right)=\\&=\binom{m}{n}\mathcal{P}\left(t^a\le t,\ t^d>t\right)^n\mathcal{P}\left(\overline{t^a\le t,\ t^d>t}\right)^{m-n}.\end{aligned}\tag{6}$$

Notice that, in (6) the probability that n vehicles are charging at time t, given that the number of daily vehicles is m, is described by the binomial probability mass function where the success probability is represented by the event that the EV charging period overlaps time t. The study of such probabilities is useful to derive the probability distribution of the energy needed by EVs at each time step. Indeed, the relation between $E^{EV}(t)$ and $H(t)$ is expressed by the following equation

$$E^{EV}(t) = H(t)\Delta P_0^{EV}.$$

Of course, the marginal cost distribution at each time step is immediately obtained from the marginal distribution of the EV energy. Unfortunately, computing the probability distribution of the EV daily cost starting from the marginal distributions of each time step is a hard task, since the energy amounts required at different time slots are correlated. In fact, to compute such distribution one needs to enumerate all the possible combinations of vehicle arrivals and departures, giving raise to a combinatorial problem whose calculation becomes intractable even for a small number of vehicles. To overcome this issue, the concept of *EV daily loss* is introduced. Let us define the EV daily loss as $\sum_{t=0}^{T-1} E^{EV}(t)(p(t)-s)$. Notice that, a negative loss means that the selling price s provides a profit. Let the selling price s and the daily electricity price profile $p(t)$ be given, we are interested in computing the following probability

$$\mathcal{P}\left(\sum_{t=0}^{T-1} E^{EV}(t)(p(t)-s) = L^{EV}\right), \tag{7}$$

where L^{EV} denotes a realization of the EV daily loss. Notice that, since s and $p(t)$ are fixed, (7) only involves discrete random variables. Computation of such a probability will be extremely important for solving Problem 1, and in particular for running Algorithm 2 which will be introduced in the next section. To compute the probability in (7), one may consider a different approach based on the probability distribution of the daily loss for a single vehicle.

For a fixed s, if vehicle v has been charged in the time interval $[\tau_1, \tau_2)$, then the corresponding loss is

$$c_v = \sum_{t=\tau_1}^{\tau_2-1} \Delta P_0^{EV}(p(t)-s).$$

Then, let us introduce the following set

$$\boldsymbol{\mathcal{C}}(h) = \Bigg\{(\tau_1,\tau_2) : \tau_1 \in [0, T-1], \tau_2 \in [0, T-1], \tau_2 > \tau_1, \\ \sum_{t=\tau_1}^{\tau_2-1} \Delta P_0^{EV}(p(t)-s) = h\Bigg\}, \tag{8}$$

that defines all the possible time windows which can lead to a loss h. Therefore, the probability that the daily loss of a vehicle is h can be computed as

$$\mathcal{P}(c_v = h) = \sum_{(\tau_1,\tau_2)\in\boldsymbol{\mathcal{C}}(h)} \mathcal{P}\left(t^a = \tau_1,\ t^d = \tau_2\right), \tag{9}$$

that is the probability that a loss h occurs is equal to the sum of the probabilities of each (τ_1, τ_2) leading to a loss h. Then, the probability of having a daily loss L^{EV} is

$$\begin{aligned}
&\mathcal{P}\left(\sum_{v=1}^{N_V} c_v = L^{EV}\right) = \\
&= \sum_{n=\underline{N}_V}^{\overline{N}_V} \mathcal{P}\left(\sum_{v=1}^{N_V} c_v = L^{EV} \middle| N_V = n\right)\mathcal{P}(N_V = n) \\
&= \sum_{n=\underline{N}_V}^{\overline{N}_V} \mathcal{P}\left(\sum_{v=1}^{n} c_v = L^{EV}\right)\mathcal{P}(N_V = n).
\end{aligned} \tag{10}$$

Notice that the single vehicle loss distributions are independent and identically distributed, since their relative arrival times and charging times are independent and identically distributed. So, the distribution of the sum of n losses is equivalent to the convolution of n single loss distributions. The previous reasoning can be summarized in the following proposition, which links (7) and (10).

Proposition 1. *The probability in* (7) *satisfies*

$$\mathcal{P}\left(\sum_{t=0}^{T-1} E^{EV}(t)(p(t)-s) = L^{EV}\right) = \mathcal{P}\left(\sum_{v=1}^{N_V} c_v = L^{EV}\right).$$

Notice that, through Proposition 1, the intractable correlation between time steps in (7) can be overcome by exploiting (10).

III. SELLING PRICE OPTIMIZATION

In this section, the chance constraint on the daily profit and the optimization problem to compute the optimal selling price are presented. To simplify the treatment, let us define the following events:

$$A \triangleq \left\{\sum_{t=0}^{T-1} E^{EV}(t)s \geq \sum_{t=0}^{T-1} E^{G}(t)p(t)\right\}, \tag{11}$$

$$B \triangleq \left\{\sum_{t=0}^{T-1} E^{EV}(t)s \geq \sum_{t=0}^{T-1} \widehat{E}^{G}(t)p(t)\right\}, \tag{12}$$

$$C \triangleq \left\{\widehat{E}^{G}(t) \geq 0,\ \forall t \in [0, T-1]\right\}. \tag{13}$$

The constraint on the selling price satisfying Problem 1 is

$$\mathcal{P}\left(\sum_{t=0}^{T-1} E^{EV}(t)s \geq \sum_{t=0}^{T-1} c(t)\right) \geq 1-\varepsilon$$

which by (2) is equivalent to

$$\mathcal{P}(A) \geq 1-\varepsilon. \tag{14}$$

Notice that, since $E^G(t)$ is defined as a *max* function involving decision variables, finding the selling price based on constraint (14) is a hard task, in general. To overcome this issue, let us introduce the following proposition which gives a sufficient condition to enforce (14).

Proposition 2. *Let*

$$\mathcal{P}(B, C) \geq 1-\varepsilon. \tag{15}$$

Then, $\mathcal{P}(A) \geq 1-\varepsilon$.

Problem 2.

$$
\begin{cases}
\min\limits_{\mathbf{P}^{ESS},s} s & & \text{(16a)}\\
\mathcal{P}\left(\begin{matrix}\sum\limits_{t=0}^{T-1} E^{EV}(t)s \geq \sum\limits_{t=0}^{T-1} \widehat{E}^{G}(t)p(t) \\ \widehat{E}^{G}(t)\geq 0,\ \forall t\in[0,T-1]\end{matrix}\right) \geq 1-\varepsilon & & \text{(16b)}\\
\widehat{E}^{G}(t) = E^{EV}(t) + \Delta P^{ESS}(t) - \widehat{E}^{PV}(t) & \forall t\in[0,T-1] & \text{(16c)}\\
0 \leq \widehat{E}^{PV}(t) \leq E^{PV}(t) & \forall t\in[0,T-1] & \text{(16d)}\\
E^{ESS}(t+1) = E^{ESS}(t) + \eta\Delta P^{ESS^{+}}(t) - \frac{1}{\eta}\Delta P^{ESS^{-}}(t) & \forall t\in[0,T-1] & \text{(16e)}\\
0 \leq E^{ESS}(t) \leq \overline{E}^{ESS} & \forall t\in[0,T] & \text{(16f)}\\
0 \leq P^{ESS^{+}}(t) \leq \overline{P}^{ESS} & \forall t\in[0,T-1] & \text{(16g)}\\
0 \leq P^{ESS^{-}}(t) \leq \overline{P}^{ESS} & \forall t\in[0,T-1] & \text{(16h)}\\
P^{ESS^{+}}(t)P^{ESS^{-}}(t) = 0 & \forall t\in[0,T-1] & \text{(16i)}\\
P^{ESS}(t) = P^{ESS^{+}}(t) - P^{ESS^{-}}(t) & \forall t\in[0,T-1] & \text{(16j)}\\
E^{ESS}(0) = E_0^{ESS} & & \text{(16k)}\\
s \geq 0. & & \text{(16l)}
\end{cases}
$$

Proof. Since $E^{G}(t) = \max\left\{\widehat{E}^{G}(t), 0\right\}$, one has $\mathcal{P}(A|C) = \mathcal{P}(B|C)$. So, $\mathcal{P}(A,C) = \mathcal{P}(A|C)\mathcal{P}(C) = \mathcal{P}(B|C)\mathcal{P}(C) = \mathcal{P}(B,C)$. Hence,

$$\mathcal{P}(A) \geq \mathcal{P}(A,C) = \mathcal{P}(B,C) \geq 1-\varepsilon. \qquad \square$$

Notice that, in Proposition 2 the event B is related to the profit, while the event C requires the energy flow to be positive throughout the day. Thanks to Proposition 2, one may introduce Problem 2 which aims to find the minimum selling price s satisfying (15).

One may notice that the nonlinearity in constraint (16i) can be substituted by replacing (16g)–(16i) with the following constraints

$$0 \leq P^{ESS-}(t) \leq \overline{P}^{ESS}(1-z_t^{ESS}) \qquad \forall t\in[0,T-1] \quad (17)$$

$$0 \leq P^{ESS+}(t) \leq \overline{P}^{ESS} z_t^{ESS} \qquad \forall t\in[0,T-1] \quad (18)$$

$$z_t^{ESS} \in \{0,1\} \qquad \forall t\in[0,T-1]. \quad (19)$$

The following proposition gives sufficient conditions to ensure (15).

Proposition 3. *Let* $0 \leq \beta \leq \varepsilon$. *If*

$$\mathcal{P}(C) \geq 1-\beta \qquad (20)$$

and

$$\mathcal{P}(B) \geq 1-\varepsilon+\beta\ , \qquad (21)$$

then $\mathcal{P}(B,C) \geq 1-\varepsilon$.

Proof. It holds that,

$$\mathcal{P}(B,C) = 1-\mathcal{P}(\overline{B}\cup\overline{C}) \geq 1-\mathcal{P}(\overline{B})-\mathcal{P}(\overline{C}) = = \mathcal{P}(B)+\mathcal{P}(C)-1 \geq 1-\varepsilon. \qquad \square$$

By (16c), probabilities in (20)–(21) can be written as

$$\mathcal{P}(C) = \mathcal{P}\left(\widehat{E}^{G}(t)\geq 0,\ \forall t\in[0,T-1]\right) = = \mathcal{P}\left(E^{EV}(t) \geq \widehat{E}^{PV}(t) - \Delta P^{ESS}(t),\ \forall t\in[0,T-1]\right),$$

and

$$\mathcal{P}(B) = \mathcal{P}\left(\sum_{t=0}^{T-1} E^{EV}(t)s \geq \sum_{t=0}^{T-1}\widehat{E}^{G}(t)p(t)\right) = = \mathcal{P}\left(\sum_{t=0}^{T-1} E^{EV}(t)(p(t)-s) \leq \sum_{t=0}^{T-1}\left(\widehat{E}^{PV}(t)-\Delta P^{ESS}(t)\right)p(t)\right). \qquad (22)$$

Notice that the probability in (22) can be effectively computed if the selling price s, the power schedule of the battery $P^{ESS}(t)$, and the energy drawn from the PV $\widehat{E}^{PV}(t)$, $\forall t\in[0,T-1]$ are known. In fact, in this case, (22) can be computed by exploiting Proposition 1 and (8)–(10).

Let us call the quantity $\sum_{t=0}^{T-1}\left(\widehat{E}^{PV}(t)-\Delta P^{ESS}(t)\right)p(t)$ the *daily savings*. In (22), it is apparent that for a given probability level, the larger the daily savings, the smaller the selling price s. Actually, Proposition 3 can be exploited to find a feasible solution of Problem 2. For this purpose, a two-step procedure is provided in the following. The first step aims to compute the maximum of the daily savings while enforcing (20). The second one, on the basis of the previously computed daily savings, is focused on finding the best selling price satisfying (21).

Focusing on the first step, for a fixed β, Problem 3 is introduced. This optimization problem aims to find the maximum daily savings r'_β guaranteeing (20) (i.e., constraint (23b) in Problem 3), that is the probability that the energy flow is negative is less than β. Notice that (23b) is a joint chance constraint. Joint chance constrained optimization problems are notoriously difficult to solve. However, such a constraint can be converted to its robust counterpart in order to derive a tractable reformulation of the original optimization problem [28]. This makes it possible to compute a solution of the original problem by employing a conservative feasible set. So, let us focus on Problem 4,

where constraint (23b) is converted to a distributionally joint chance constraint where the probability distribution $\mathcal{P}$ belongs

Problem 3.

$$
\begin{cases}
r'_\beta = \max\limits_{\mathbf{P}^{ESS}} \sum\limits_{t=0}^{T-1} \left(\widehat{E}^{PV}(t) - \Delta P^{ESS}(t)\right) p(t) & & \text{(23a)}\\
\quad \mathcal{P}\left(E^{EV}(t) \geq \widehat{E}^{PV}(t) - \Delta P^{ESS}(t),\ \forall t \in [0, T-1]\right) \geq 1-\beta & & \text{(23b)}\\
\quad 0 \leq \widehat{E}^{PV}(t) \leq E^{PV}(t) & \forall t \in [0, T-1] & \text{(23c)}\\
\quad E^{ESS}(t+1) = E^{ESS}(t) + \eta \Delta P^{ESS^+}(t) - \frac{1}{\eta}\Delta P^{ESS^-}(t) & \forall t \in [0, T-1] & \text{(23d)}\\
\quad 0 \leq E^{ESS}(t) \leq \overline{E}^{ESS} & \forall t \in [0, T] & \text{(23e)}\\
\quad P^{ESS}(t) = P^{ESS^+}(t) - P^{ESS^-}(t) & \forall t \in [0, T-1] & \text{(23f)}\\
\quad 0 \leq P^{ESS-}(t) \leq \overline{P}^{ESS}(1 - z_t^{ESS}) & \forall t \in [0, T-1] & \text{(23g)}\\
\quad 0 \leq P^{ESS+}(t) \leq \overline{P}^{ESS} z_t^{ESS} & \forall t \in [0, T-1] & \text{(23h)}\\
\quad z_t^{ESS} \in \{0, 1\} & \forall t \in [0, T-1] & \text{(23i)}\\
\quad E^{ESS}(0) = E_0^{ESS}. & & \text{(23j)}
\end{cases}
$$

Problem 4.

$$
\begin{cases}
r_\beta = \max\limits_{\mathbf{P}^{ESS}} \sum\limits_{t=0}^{T-1} \left(\widehat{E}^{PV}(t) - \Delta P^{ESS}(t)\right) p(t) & & \text{(24a)}\\
\quad \inf\limits_{\mathcal{P}\in\mathcal{D}} \mathcal{P}\left(E^{EV}(t) \geq \widehat{E}^{PV}(t) - \Delta P^{ESS}(t),\ \forall t \in [0, T-1]\right) \geq 1-\beta & & \text{(24b)}\\
\quad 0 \leq \widehat{E}^{PV}(t) \leq E^{PV}(t) & \forall t \in [0, T-1] & \text{(24c)}\\
\quad E^{ESS}(t+1) = E^{ESS}(t) + \eta \Delta P^{ESS^+}(t) - \frac{1}{\eta}\Delta P^{ESS^-}(t) & \forall t \in [0, T-1] & \text{(24d)}\\
\quad 0 \leq E^{ESS}(t) \leq \overline{E}^{ESS} & \forall t \in [0, T] & \text{(24e)}\\
\quad P^{ESS}(t) = P^{ESS^+}(t) - P^{ESS^-}(t) & \forall t \in [0, T-1] & \text{(24f)}\\
\quad 0 \leq P^{ESS-}(t) \leq \overline{P}^{ESS}(1 - z_t^{ESS}) & \forall t \in [0, T-1] & \text{(24g)}\\
\quad 0 \leq P^{ESS+}(t) \leq \overline{P}^{ESS} z_t^{ESS} & \forall t \in [0, T-1] & \text{(24h)}\\
\quad z_t^{ESS} \in \{0, 1\} & \forall t \in [0, T-1] & \text{(24i)}\\
\quad E^{ESS}(0) = E_0^{ESS}, & & \text{(24j)}
\end{cases}
$$

to the ambiguity set $\mathcal{D}$. Such an ambiguity set is defined as follows

$$\mathcal{D} = \left\{\mathcal{P} : \mathcal{P} \in \mathcal{M}(\mathcal{P}_0, \ldots, \mathcal{P}_{T-1}), E^{EV}(t) \sim \mathcal{P}_t, \forall t \in [0, T-1]\right\}$$

where $\mathcal{P}_t$ denotes the probability distribution of $E^{EV}(t)$, and $\mathcal{M}(\mathcal{P}_0, \ldots, \mathcal{P}_{T-1})$ denotes the set of all joint probability distributions whose marginals are $\mathcal{P}_t$. As described in [28], since random variables $E^{EV}(t)$ are distributed along discrete supports, Problem 4 is equivalent to the mixed integer linear program (MILP) reported in Problem 5. In this problem, the original robust joint chance constraint (24b) has been reformulated by using the Bonferroni inequality, where the optimizer is allowed to choose the best option for the Bonferroni weights β_t. Concerning the binary variables $z_{t,n}^{EV}$, they are used to satisfy each individual chance constraint (25c).

Remark 1. *Notice that, the number of binary variables $z_{t,n}^{EV}$ in Problem 5 may be large, since it equals $T(\overline{N}_V + 1)$. Thus, the computational burden of Problem 5 might be intractable, in general. However, constraint* (25n) *enforces that only one binary variable can be equal to* 1 *at each time step, making the problem structure suitable to be efficiently managed by standard solvers, as reported in Section V.*

In Problem 5, the maximum daily savings r_β for a fixed β is computed. The second step of the procedure concerns the computation of the best selling price s_β for given β and r_β. Such a selling price can be obtained by solving Problem 6, where constraint (26b) coincides with (21) for a given amount of daily savings r_β, i.e. by considering the optimal solution of Problem 5.

Problem 6 (min_selling_price)**.**

$$
\begin{cases}
s_\beta = \min\ s & \text{(26a)}\\
\quad \mathcal{P}\left(\sum\limits_{t=0}^{T-1} E^{EV}(t)(p(t) - s) \leq r_\beta\right) \geq 1 - \varepsilon + \beta & \text{(26b)}\\
\quad s \geq 0. & \text{(26c)}
\end{cases}
$$

A. Optimal selling price algorithm

To compute the optimal selling price satisfying Problem 1, the procedure reported in Algorithm 1 has been devised.

First, initialization of some variables are performed. In particular, the candidate optimal selling price s^* is set to the maximum of the grid electricity price, while the candidate optimal value of β is set to 0. Since $0 \leq \beta < \varepsilon$, a grid search in β with step γ from 0 to $\varepsilon - \gamma$ is performed. For any value of β, the subroutine *max_savings* provides the solution of Problem 5 returning the related daily savings r_β, while *min_selling_price* returns s_β, i.e. the solution of Problem 6. If

Problem 5 (max_savings)**.**

$$
\begin{cases}
r_\beta = \max_{\mathbf{P}^{ESS}} \sum_{t=0}^{T-1} \left(\widehat{E}^{PV}(t) - \Delta P^{ESS}(t)\right) p(t) & & \text{(25a)}\\
\sum_{n=0}^{\overline{N}_V} \left(\Delta P_0^{EV} n\right) z_{t,n}^{EV} \geq \widehat{E}^{PV}(t) - \Delta P^{ESS}(t) & \forall t \in [0, T-1] & \text{(25b)}\\
\sum_{n=0}^{\overline{N}_V} \mathcal{P}_t \left(E^{EV}(t) \geq \Delta P_0^{EV} n\right) z_{t,n}^{EV} \geq 1 - \beta_t & \forall t \in [0, T-1] & \text{(25c)}\\
0 \leq \widehat{E}^{PV}(t) \leq E^{PV}(t) & \forall t \in [0, T-1] & \text{(25d)}\\
E^{ESS}(t+1) = E^{ESS}(t) + \eta \Delta P^{ESS^+}(t) - \frac{1}{\eta} \Delta P^{ESS^-}(t) & \forall t \in [0, T-1] & \text{(25e)}\\
0 \leq E^{ESS}(t) \leq \overline{E}^{ESS} & \forall t \in [0, T] & \text{(25f)}\\
P^{ESS}(t) = P^{ESS^+}(t) - P^{ESS^-}(t) & \forall t \in [0, T-1] & \text{(25g)}\\
0 \leq P^{ESS-}(t) \leq \overline{P}^{ESS}(1 - z_t^{ESS}) & \forall t \in [0, T-1] & \text{(25h)}\\
0 \leq P^{ESS+}(t) \leq \overline{P}^{ESS} z_t^{ESS} & \forall t \in [0, T-1] & \text{(25i)}\\
z_t^{ESS} \in \{0, 1\} & \forall t \in [0, T-1] & \text{(25j)}\\
E^{ESS}(0) = E_0^{ESS} & & \text{(25k)}\\
\beta_t \geq 0 & \forall t \in [0, T-1] & \text{(25l)}\\
z_{t,n}^{EV} \in \{0, 1\} & \forall t \in [0, T-1],\ n \in [0, \overline{N}_V] & \text{(25m)}\\
\sum_{n=0}^{\overline{N}_V} z_{t,n}^{EV} = 1 & \forall t \in [0, T-1] & \text{(25n)}\\
\sum_{t=0}^{T-1} \beta_t \leq \beta. & & \text{(25o)}
\end{cases}
$$

Data: ε, $p(t)$, E_0^{ESS}, α, distributions on t_a, t_c, N_V, and step size γ

$s^* = \max_{t \in [0, T-1]} p(t)$;
$\beta^* = 0$;
for $\beta = 0$ **to** $(\varepsilon - \gamma)$ **by** γ **do**
  $r_\beta = max\_savings(\beta)$;
  $s_\beta = min\_selling\_price(\beta, r_\beta)$;
  if $s_\beta < s^*$ **then**
    $s^* = s_\beta$;
    $\beta^* = \beta$;
  end
end
return s^*, β^*;

Algorithm 1: Main procedure to solve Problem 1.

the current selling price is less than the current optimal value, s^* and β^* are updated accordingly. Finally, the optimal values are returned.

Since Problem 5 is a MILP, it can be easily implemented in the *max_savings* subroutine. On the other hand, the solution of Problem 6 requires an ad-hoc procedure. Notice that, Problem 6 involves an individual chance constraint where the only decision variable is related to the selling price s. Moreover, the probability on the left hand side of constraint (26b) is monotonically non-decreasing with respect to s, and so a bisection procedure on s can be designed, see Algorithm 2. The lowest value of the selling price has been set to $s_{bot} = 0$, while the highest one coincides with the maximum of the electricity price over the day, i.e. $s_{top} = \max_{t \in [0, T-1]} p(t)$, which guarantees a non-negative profit. The computation of ε_{top}, ε_{bot}, and ε_{mid} is performed by exploiting Proposition 1

Data: β, r_β, ε, $p(t)$, distributions on t_a, t_c, N_V and tolerance γ_ε

$s_{bot} = 0$; $s_{top} = \max_{t \in [0, T]} p(t)$; $\varepsilon_{sat} = 1 - \varepsilon + \beta$;
$\varepsilon_{bot} = \mathcal{P}\left(\sum_{t=0}^{T-1} E^{EV}(t)(p(t) - s_{bot}) \leq r_\beta\right)$;
$\varepsilon_{top} = \mathcal{P}\left(\sum_{t=0}^{T-1} E^{EV}(t)(p(t) - s_{top}) \leq r_\beta\right)$;
while $\frac{\varepsilon_{top} - \varepsilon_{sat}}{\varepsilon_{sat}} > \gamma_\varepsilon$ **do**
  $s_{mid} = \frac{s_{top} + s_{bot}}{2}$;
  $\varepsilon_{mid} = \mathcal{P}\left(\sum_{t=0}^{T-1} E^{EV}(t)(p(t) - s_{mid}) \leq r_\beta\right)$;
  if $\varepsilon_{mid} \geq \varepsilon_{sat}$ **then**
    $\varepsilon_{top} = \varepsilon_{mid}$;
    $s_{top} = s_{mid}$;
  else
    $\varepsilon_{bot} = \varepsilon_{mid}$;
    $s_{bot} = s_{mid}$;
  end
end
return s_{top};

Algorithm 2: Bisection procedure to solve Problem 6 for given β and r_β.

and (8)–(10). The algorithm stops when the relative difference between the maximum and the minimum probability level in (26b) is less than a given tolerance γ_ε. Because of the bisection formulation, computation of *min_selling_price* is efficiently performed.

Remark 2. *To help the reader, the complete reasoning underpinning the proposed method is summarized. In particular, a summary showing that Algorithm 1, where Problem 5 and Problem 6 are sequentially solved, provides a feasible solution of Problem 1 is outlined in Fig. 1. To clarify the scheme*

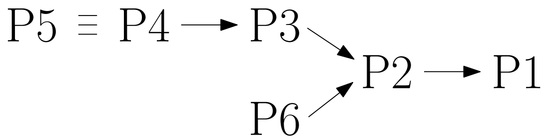

Fig. 1. Functional relationship representation of the procedure for solving Problem 1. Notation P1, . . . , P6 stands for Problem 1, . . . , Problem 6.

reported in Fig. 1, the following statements are in order:

- *an optimal solution of Problem 5 is optimal for the equivalent Problem 4;*
- *an optimal solution of Problem 5 provides a feasible estimate of the optimal cost function of Problem 3;*
- *by Proposition 3, the optimal solutions of Problem 5 and Problem 6 provide a feasible approximation of Problem 2;*
- *by Proposition 2, the solution of Problem 2 is a feasible approximation of the original Problem 1.*

IV. Optimal ESS operation

The above mentioned method is used to compute the optimal selling price at the beginning of the day. However, during the day, the energy schedule of the ESS may be adapted to further improve the daily profit, taking into account the actual realizations of the stochastic processes modeling the EVs. In fact, at each time step, it is reasonable to assume the knowledge of the energy to be charged in each connected vehicle and their departure times, as well as the number of EVs that have arrived up to the present time. A receding horizon procedure to operate the ESS during the course of the day can be designed, following a similar reasoning to that described in Section III.

Let us denote the present time step by τ, and let $H(\tau, t)$ be the number of connected vehicles which will be still in charge at time $t > \tau$, i.e. $H(\tau,t) = \left|\left\{v : t_v^a \leq \tau,\ t_v^d > t\right\}\right|$, where $|\cdot|$ denotes the cardinality operator of a set. Notice that, at time τ, $H(\tau,t)$ is known for any $t > \tau$.

Let $N_V(0,\tau)$ be the random variable associated to the number of incoming vehicles during the interval $[0,\tau]$, and let us denote by n_a the actual number of vehicles arrived up to the present time step, i.e. in the interval $[0,\tau]$. So, the distribution of the daily incoming vehicles can be refined by exploiting the knowledge of the already arrived vehicles n_a,

$$\begin{aligned}&\mathcal{P}\left(N_V = n | N_V(0,\tau) = n_a\right) = \\ &= \frac{\mathcal{P}\left(N_V(0,\tau) = n_a | N_V = n\right)\mathcal{P}\left(N_V = n\right)}{\mathcal{P}\left(N_V(0,\tau) = n_a\right)},\end{aligned} \tag{27}$$

where

$$\mathcal{P}(N_V(0,\tau) = n_a) = \sum_{k=n_a}^{\overline{N}_V} \mathcal{P}(N_V(0,\tau) = n_a | N_V = k)\, \mathcal{P}(N_V = k)$$

and

$$\begin{aligned}&\mathcal{P}\left(N_V(0,\tau) = n_a | N_V = k\right) = \\ &= \binom{k}{n_a} \mathcal{P}\left(0 \leq t_a \leq \tau\right)^{n_a} \mathcal{P}\left(\overline{0 \leq t_a \leq \tau}\right)^{k-n_a}.\end{aligned}$$

Data: ε, $p(t)$, E_0^{ESS}, and distributions on t_a, t_c, N_V
Set $\beta = \beta^*$ as returned in Algorithm 1;
Set $\tau = 0$ and $E_0^{ESS}(0) = E_0^{ESS}$;
while $\tau < T - 1$ **do**
  $n_a = |\{v : t_v^a \leq \tau\}|$;
  $H(\tau,t) = |\{v : t_v^a \leq \tau, t_v^d \geq t\}|\ ,\ \forall t > \tau$;
  compute $P(N_V = n | N_V(0,\tau) = n_a),\ \forall n \in [n_a, \overline{N}_v]$ as in (27);
  compute $P(t_a = t | t_a > \tau),\ \forall t \in [\tau+1, T-1]$ as in (28);
  compute distributions of $\widetilde{H}(t), \forall t \in [\tau+1, T-1]$;
  solve Problem 5 by using the updated constraints (29)–(30), and get $P^{ESS}(\tau)$;
  $E_0^{ESS}(\tau+1) = E_0^{ESS}(\tau) + \eta\Delta P^{ESS^*+}(\tau) - \frac{1}{\eta}\Delta P^{ESS^*-}(\tau)$;
  $\tau = \tau + 1$;
end

Algorithm 3: Receding horizon algorithm.

For $t > \tau$, the arrival time distribution can be updated as

$$\mathcal{P}(t^a = t | t^a > \tau) = \frac{\mathcal{P}(t^a > \tau | t^a = t)\mathcal{P}(t^a = t)}{\mathcal{P}(t^a > \tau)} = \frac{\mathcal{P}(t^a = t)}{\mathcal{P}(t^a > \tau)}. \tag{28}$$

Let us denote by $\widetilde{H}(t)$ the random variable describing the number of vehicles in charge at time $t > \tau$, ignoring the EVs which are in charge at time τ. In other words, $\widetilde{H}(t)$ coincides with $H(t)$ if one assumes empty parking lots and initial time step equal to τ. Then, the probability distribution of $\widetilde{H}(t)$ can be computed similarly to that of $H(t)$ in (5), by using the updated distributions of the total number of incoming vehicles and the EV arrival times, i.e. (27) and (28), respectively.

To exploit the same reasoning described in Section III, the receding horizon procedure is based on the solution of Problem 5, where the optimization is performed in $[\tau, T-1]$, and constraints (25b)–(25c) are adjusted as follows

$$\sum_{n=0}^{\overline{N}_V} \left(\Delta P_0^{EV} n\right) z_{t,n}^{EV} \geq \widehat{E}^{PV}(t) - \Delta P^{ESS}(t) - H(\tau,t)\Delta P_0^{EV}, \quad \forall t \in [\tau, T-1] \tag{29}$$

$$\sum_{n=0}^{\overline{N}_V} \mathcal{P}_t\left(\widetilde{E}^{EV}(t) \geq \Delta P_0^{EV} n\right) z_{t,n}^{EV} \geq 1 - \beta_t,\ \forall t \in [\tau, T-1], \tag{30}$$

where $\widetilde{E}^{EV}(t) = \Delta P_0^{EV} \widetilde{H}(t)$. A sketch of the receding horizon procedure to online operate the ESS is given in Algorithm 3.

V. Numerical simulations

In order to validate the proposed procedure, numerical simulations have been performed. It is assumed that the charging station is located in a commercial center and it supplies energy from 4:00 till 24:00. The sampling time is set to 10 minutes, i.e. $\Delta = 1/6$ hours, while the nominal charging power for electric vehicles is 22 kW. The ESS capacity is 1000 kWh, while its maximum charging and discharging power rates are 500 kW, with an efficiency $\eta = 0.9$. Concerning the PV plant, its peak power is assumed equal to 90 kW. To perform simulations, for each day the PV production has been scaled by a uniform distribution in the interval $[0.2, 1]$. An example of PV production realization over 10 days is depicted in Fig. 2. Figure 3 shows the EV arrival time distribution, and

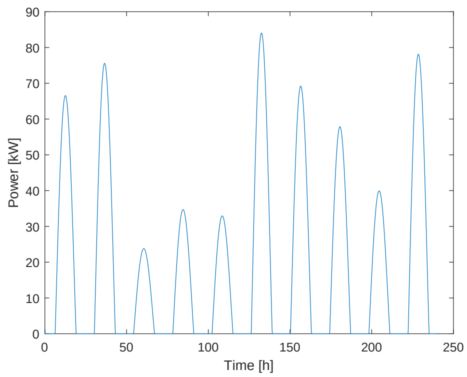

Fig. 2. Photovoltaic production in 10 simulated days.

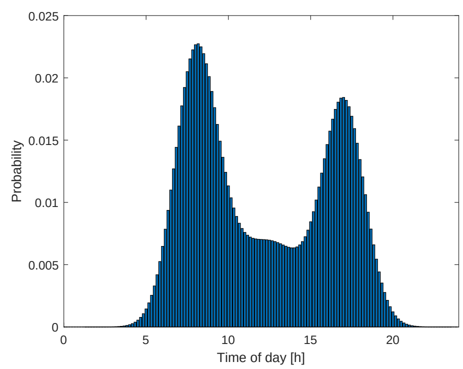

Fig. 3. Arrival time distribution of incoming EVs.

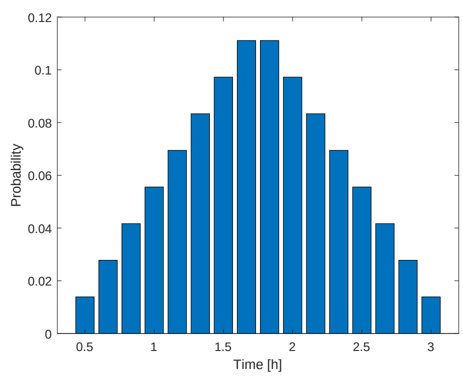

Fig. 4. EV charging time distribution.

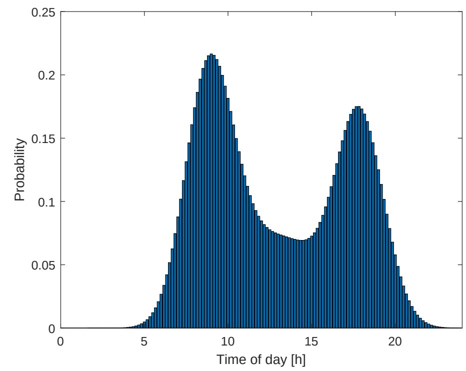

Fig. 5. Probability of having a generic vehicle in charge at a fixed time.

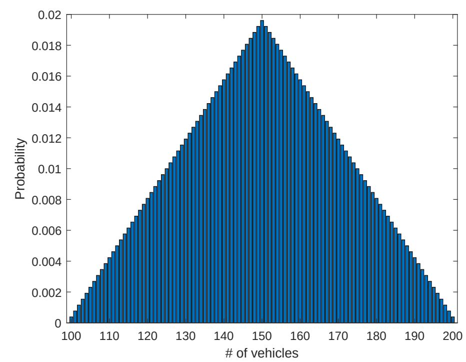

Fig. 6. Probability distribution of the total number of vehicles for each day.

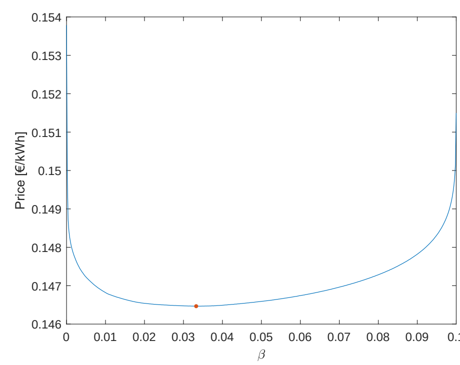

Fig. 7. Value of the selling price s_β with respect to β (day 93). The minimum selling price s^* and the corresponding β^* are denoted by the red dot.

the probability distribution of the charging time is given in Fig. 4. By computing the convolution in (4), it is possible to obtain the probabilities that a generic vehicle is in charge at a fixed time, shown in Fig. 5.

The number of daily incoming vehicles has been modeled by a symmetric triangular distribution with $\underline{N}_V = 100$ and $\overline{N}_V = 200$, as shown in Fig. 6. Concerning the daily profit, the fraction α has been set to 0.2, corresponding to an earnings/cost ratio of 1.2, while the probability level ε is 10%. The electricity price has been taken from the Italian electricity market [29], where for each simulated day a new electricity price profile is considered.

The configuration described above has been simulated for 300 days. In Fig. 7, the relation between β and the selling price s_β for a generic day is depicted. Notice that in Problem 6 it is not easy to figure out how s_β evolves with β, since it also depends on r_β which is again a function of β. In the reported day, the optimal value of s_β is obtained when β is 3.33%. To validate the satisfaction of constraint (15), the computed selling price and the battery schedule obtained by using Algorithm 1 have been considered. In this simulation, the number of days where this constraint is not satisfied amounts to 15, corresponding to 5% of the total.

The average selling price lies between the mean and the maximum value of the daily electricity price, as shown in the box plots of Fig. 8. In Fig. 9, the daily electricity price and daily selling price for day 93 are shown. Photovoltaic production and overall energy needed by the EVs for the same

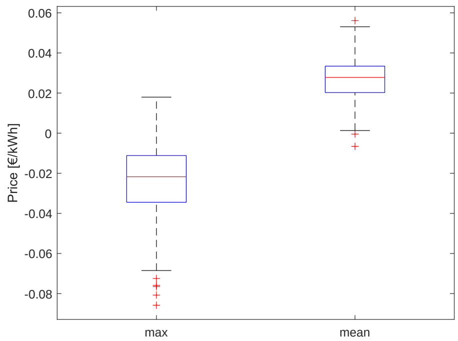

Fig. 8. Left: Boxplot of the difference between the selling price and the maximum of the energy price. Right: Boxplot of the difference between the selling price and the mean of the energy price.

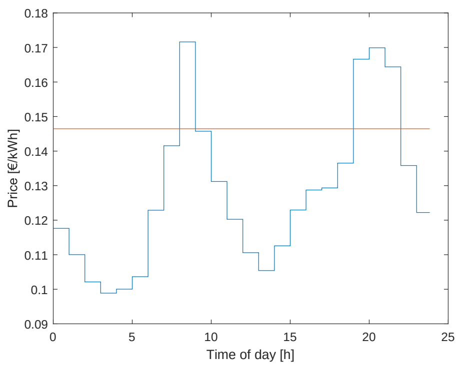

Fig. 9. Daily electricity price (blue) and optimal selling price (red), for day 93.

day are reported in Fig. 10.

To assess the performance of the receding horizon battery schedule, a comparison with respect to the one computed at the beginning of the day has been carried out. The ESS power schedules $P^{ESS}(t)$ in day 93 for both the initial and the receding horizon strategies are shown in Fig. 11 [top], while the corresponding levels of charge are reported in Fig. 11 [bottom]. Notice that, during the first time period, both solutions almost coincide because no vehicles have yet arrived. However, when vehicles start arriving to the parking lot, the two profiles separate. Notice that the behavior of the ESS charging/discharging power has a smooth profile throughout the day when computed at the beginning of the day, while it is sharper in some intervals for the receding horizon implementation. This is not surprising since the former is computed open-loop on the basis of the given distributions, while the latter is recursively computed at each time step on the basis of the actual outcomes of the stochastic processes. As a result of the knowledge of the past and actual vehicle arrivals and energy required, the receding horizon strategy outperforms the one computed at the beginning of the day, providing an earnings/cost ratio of 1.24 versus 1.22.

In Fig. 12, the daily earnings/cost ratio difference between the initial and receding horizon plans is depicted, while in Fig. 13 the earnings/cost ratio boxplot for both procedures are shown. As expected, in general, the receding horizon strategy outperforms the one computed at the beginning of the day. In fact, since Algorithm 1 is executed once at the beginning of the day, the computed control actions depend on a priori information and not on the actual realizations of the random variables. On the contrary, the receding horizon implementation exploits past and present vehicle information as it works in a closed loop manner. So, it is very likely that the receding horizon algorithm outperforms Algorithm 1, even if this is not guaranteed in general. Moreover, the number of days when constraint (15) is violated has been reduced to 2.

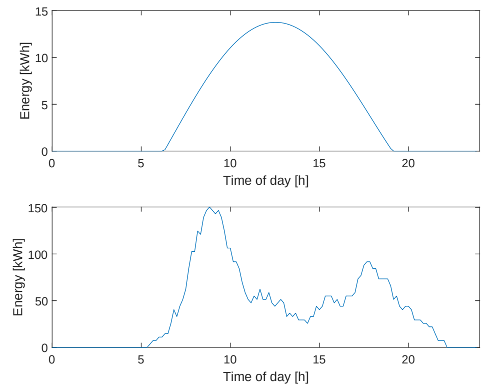

Fig. 10. Day 93. PV production [top] and energy needed to charge EVs [bottom].

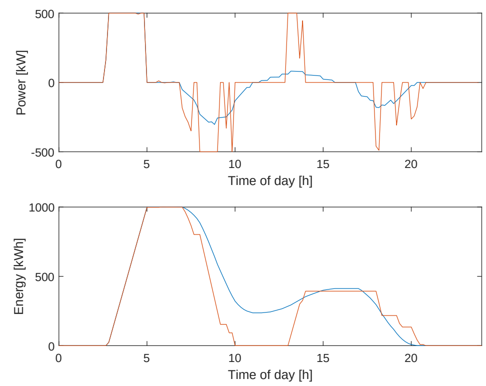

Fig. 11. Day 93. ESS power schedule [top] and corresponding level of charge [bottom] computed by Algorithm 1 (blue) and Algorithm 3 (red).

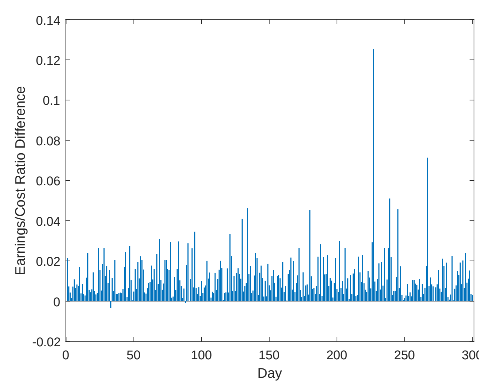

Fig. 12. Difference between the earnings/cost ratio obtained by the receding horizon procedure and that computed by Algorithm 1 at the beginning of a day.

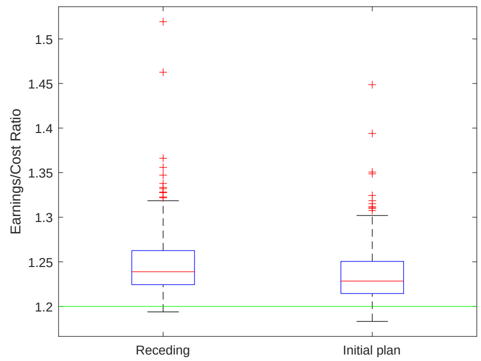

Fig. 13. Boxplot of the earnings/cost ratio for the receding horizon EDS schedule (left) and the policy computed at the beginning of the day (right). The green line denotes the specified probabilistic threshold.

A. Discussion

It is worth noting that the proposed technique exploits the EV distributions without requiring limiting assumptions on them. This means that the approach allows one to deal with different daily scenarios, possibly accounting for weekdays, holidays, strikes, etc.

Examining Fig. 13 (right), one can see that in general the selling price computed by Algorithm 1 at the beginning of the day leads to an earnings/cost ratio greater than the required threshold. This result fully agrees with the requirements. In fact, the small number of days falling below the threshold are less than the prescribed 10%-limit, but at the same time, it is greater than zero, showing a low degree of conservativeness.

Once the selling price has been fixed, the proposed receding horizon algorithm can be used to operate the EDS during the day. This procedure takes advantage of the knowledge of the random processes outcomes related to EVs, which are clearly known only in probabilistic terms at the beginning of the day when Algorithm 1 is run. So, it is expected that the daily profit obtained by the receding horizon procedure outperforms that given by operating the ESS with the commands provided at the beginning of the day. This has been confirmed by numerical simulations as reported in Fig. 12, where the difference between the daily earnings/cost ratio provided by the two procedures is shown. In addition, the receding horizon algorithm shows better performance in terms of violations of constraint (15).

Regarding the computation of the best value of β, note that in order to obtain the best selling price, the optimal value of β changes day by day on the basis of the expected PV production and of the electricity price profile. Moreover, as shown in Fig. 7, the flat behavior of the selling price around the optimal value β^*, allows one to perform the grid search in Algorithm 1 with only a few iterations.

Concerning computational aspects, Algorithm 2 needs about 20 iterations to return the optimal value of Problem 6, with a tolerance $\gamma_\varepsilon = 10^{-6}$, while the loop in Algorithm 1 has been run over 100 equally spaced values between 0 and ε, i.e. with a resolution of 10^{-3}. Such a resolution is enough to obtain a value of s_β very close to the optimal one (see Fig. 7). For this problem, Algorithm 1 takes on average 3.5 minutes to compute the optimal selling price, while the receding horizon strategy performs one iteration step in 0.18 seconds. Simulations have been run in MATLAB, with the optimization problems formulated using YALMIP [30] and solved by CPLEX [31] on an Intel(R) Core(TM) i7-7700 CPU @3.60 GHz with 32 GB of RAM. Thanks to the low computational burden, both algorithms can be effectively used in practical applications even with a larger number of incoming vehicles.

VI. Conclusions

To derive an effective energy pricing strategy under uncertain EV demand, a chance constrained optimization problem has been formulated. The resulting chance constraint is based on probability distributions that characterize the EV daily behavior. A tractable formulation of the original problem has been derived, and a procedure to compute the optimal selling price has been devised. While the selling price is computed at the beginning of the day, during the day a receding horizon strategy to operate the ESS, by exploiting the realizations of the EV random processes, has been designed. Simulation results show that such approach is close to optimal. In fact, on average, the daily profit is reasonably close to the required value, while the chance constraint is tightly satisfied. The receding horizon strategy to adapt the battery charging schedule generally leads to better performance. Moreover, the number of chance constraint violations is reduced.

Future developments will address more complex scenarios, where uncertainty on renewable forecasts and on electricity price are explicitly taken into account. Moreover, charging stations equipped with vehicle-to-grid facilities can be considered, as well as the participation of the parking lot in demand response programs. An additional research direction is related to competitive environments, where the energy pricing problem will be analyzed by considering the presence of other players involved in the charging service.

References

[1] Q. Kang, J. Wang, M. Zhou, and A. C. Ammari, "Centralized charging strategy and scheduling algorithm for electric vehicles under a battery swapping scenario," *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 3, pp. 659–669, 2016.

[2] O. Erdinç, A. Taşçıkaraoğlu, N. G. Paterakis, İ. Dursun, M. C. Sinim, and J. Catalão, "Comprehensive optimization model for sizing and siting of DG units, EV charging stations, and energy storage systems," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 3871–3882, 2018.

[3] Y. Xiong, J. Gan, B. An, C. Miao, and A. L. C. Bazzan, "Optimal electric vehicle fast charging station placement based on game theoretical framework," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 8, pp. 2493–2504, 2018.

[4] M. Maigha and M. L. Crow, "A transactive operating model for smart airport parking lots," *IEEE Power and Energy Technology Systems Journal*, vol. 5, no. 4, pp. 157–166, 2018.

[5] L. Zhang and Y. Li, "A game-theoretic approach to optimal scheduling of parking-lot electric vehicle charging," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 6, pp. 4068–4078, 2015.

[6] W. Jiang and Y. Zhen, "A real-time EV charging scheduling for parking lots with PV system and energy store system," *IEEE Access*, vol. 7, pp. 86 184–86 193, 2019.

[7] M. Ş. Kuran, A. C. Viana, L. Iannone, D. Kofman, G. Mermoud, and J. P. Vasseur, "A smart parking lot management system for scheduling the recharging of electric vehicles," *IEEE Transactions on Smart Grid*, vol. 6, no. 6, pp. 2942–2953, 2015.

[8] M. Casini, G. G. Zanvettor, M. Kovjanic, and A. Vicino, "Optimal energy management and control of an industrial microgrid with plug-in electric vehicles," *IEEE Access*, vol. 7, pp. 101 729–101 740, 2019.

[9] X. Tan, G. Qu, B. Sun, N. Li, and D. H. K. Tsang, "Optimal scheduling of battery charging station serving electric vehicles based on battery swapping," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 1372–1384, 2019.

[10] M. R. Sarker, H. Pandžić, and M. A. Ortega-Vazquez, "Optimal operation and services scheduling for an electric vehicle battery swapping station," *IEEE Transactions on Power Systems*, vol. 30, no. 2, pp. 901–910, 2015.

[11] B. Hashemi, M. Shahabi, and P. Teimourzadeh-Baboli, "Stochastic-based optimal charging strategy for plug-in electric vehicles aggregator under incentive and regulatory policies of DSO," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 4, pp. 3234–3245, April 2019.

[12] A. Alsabbagh, X. Wu, and C. Ma, "Distributed electric vehicles charging management considering time anxiety and customer behaviors," *IEEE Transactions on Industrial Informatics*, pp. 1–1, 2020.

[13] M. Casini, A. Vicino, and G. G. Zanvettor, "A distributionally robust joint chance constraint approach to smart charging of plug-in electric vehicles," in *Proc. 58th IEEE Conf. on Decision and Control*, Nice, December 2019, pp. 4222–4227.

[14] W. Tang and Y. J. A. Zhang, "A model predictive control approach for low-complexity electric vehicle charging scheduling: Optimality and scalability," *IEEE transactions on power systems*, vol. 32, no. 2, pp. 1050–1063, 2016.

[15] M. Casini, A. Vicino, and G. G. Zanvettor, "A receding horizon approach to peak power minimization for EV charging stations in the presence of uncertainty," *International Journal of Electrical Power & Energy Systems*, vol. 126, p. 106567, 2021.

[16] G. S. Aujla, N. Kumar, M. Singh, and A. Y. Zomaya, "Energy trading with dynamic pricing for electric vehicles in a smart city environment," *Journal of Parallel and Distributed Computing*, vol. 127, pp. 169 – 183, 2019.

[17] M. Alizadeh, H. Wai, M. Chowdhury, A. Goldsmith, A. Scaglione, and T. Javidi, "Optimal pricing to manage electric vehicles in coupled power and transportation networks," *IEEE Transactions on Control of Network Systems*, vol. 4, no. 4, pp. 863–875, 2017.

[18] Z. Moghaddam, I. Ahmad, D. Habibi, and M. A. Masoum, "A coordinated dynamic pricing model for electric vehicle charging stations," *IEEE Transactions on Transportation Electrification*, vol. 5, no. 1, pp. 226–238, 2019.

[19] V. Subramanian and T. K. Das, "A two-layer model for dynamic pricing of electricity and optimal charging of electric vehicles under price spikes," *Energy*, vol. 167, pp. 1266 – 1277, 2019.

[20] S. Limmer and T. Rodemann, "Peak load reduction through dynamic pricing for electric vehicle charging," *International Journal of Electrical Power & Energy Systems*, vol. 113, pp. 117 – 128, 2019.

[21] J. Soares, M. A. F. Ghazvini, N. Borges, and Z. Vale, "Dynamic electricity pricing for electric vehicles using stochastic programming," *Energy*, vol. 122, pp. 111 – 127, 2017.

[22] Z. Hu, K. Zhan, H. Zhang, and Y. Song, "Pricing mechanisms design for guiding electric vehicle charging to fill load valley," *Applied Energy*, vol. 178, pp. 155–163, 2016.

[23] N. Y. Soltani, S.-J. Kim, and G. B. Giannakis, "Real-time load elasticity tracking and pricing for electric vehicle charging," *IEEE Transactions on Smart Grid*, vol. 6, no. 3, pp. 1303–1313, 2014.

[24] Z. Li, Q. Guo, H. Sun, S. Xin, and J. Wang, "A new real-time smart-charging method considering expected electric vehicle fleet connections," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 3114–3115, 2014.

[25] A. Ghosh and V. Aggarwal, "Menu-based pricing for profitable electric vehicle charging with vehicle-to-grid service," in *2018 International Conference on Signal Processing and Communications (SPCOM)*, July 2018, pp. 237–241.

[26] T. M. Aljohani, A. F. Ebrahim, and O. A. Mohammed, "Dynamic real-time pricing mechanism for electric vehicles charging considering optimal microgrids energy management system," *IEEE Transactions on Industry Applications*, vol. 57, no. 5, pp. 5372–5381, 2021.

[27] C. Luo, Y. Huang, and V. Gupta, "Stochastic dynamic pricing for EV charging stations with renewable integration and energy storage," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 1494–1505, March 2018.

[28] W. Xie, S. Ahmed, and R. Jiang, "Optimized Bonferroni approximations of distributionally robust joint chance constraints," *Mathematical Programming*, pp. 1–34, 2019.

[29] Gestore Mercati Energetici, "Andamento dei prezzi e dei volumi," 2019, online: http://www.mercatoelettrico.org.

[30] J. Löfberg, "Yalmip : A toolbox for modeling and optimization in Matlab," in *In Proc. of the CACSD Conference*, Taipei, Taiwan, 2004.

[31] IBM, "IBM ILOG Cplex Optimizer," online: https://www.ibm.com/products/ilog-cplex-optimization-studio.